Economics 316

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Problem Set 5

1. Consider the mixed strategy Nash equilibrium of the model of expert diagnosis studied in class. How does the equilibrium change if *both* E and E' decrease by the same amount (a major repair becomes less costly for both an expert and a consumer)? How does the equilibrium change if both E' and I' decrease (both do-it-yourself repairs become less costly)?

2. In the model of expert diagnosis presented in class, suppose that some experts always report honestly (because they regard doing so as the only morally acceptable action, for example). (Virág Fórizs, a student in the class in Winter 2013, posed this question.)

   Suppose that there are of two types of experts. The fraction α always report problems honestly—call these experts *principled*. The remaining fraction $1 - \alpha$ are *unprincipled*, as in the original model: they choose whichever action *Honest* or *Dishonest* yields them a higher expected payoff. Assume that a consumer does not know whether the expert with whom they are dealing is principled or unprincipled—she observes only the expert's recommendation. Thus the only experts who are "players" in the game are the unprincipled ones. If the fraction p of them choose *Honest*, then the probability of a consumer being matched with an expert who chooses *Honest* is $\alpha + (1 - \alpha)p$.

   Find the mixed strategy Nash equilibrium (equilibria?) in this case. The nature of the equilibrium depends on whether $\alpha < p^*$, $\alpha = p^*$, or $\alpha > p^*$, where p^* is equilibrium probability that the expert is honest in the original model.

3. Each of two sellers has available one indivisible unit of a good. Seller 1 posts the price p_1 and seller 2 posts the price p_2. Each of two buyers would like to obtain one unit of the good; they simultaneously decide which seller to approach. If both buyers approach the same seller, each trades with probability $\frac{1}{2}$; the disappointed buyer does not subsequently have the option to trade with the other seller. (This assumption models the risk faced by a buyer that a good is sold out

	Stop	*Continue*
Stop	$1, 1$	$1-\varepsilon, 2$
Continue	$2, 1-\varepsilon$	$0, 0$

Figure 1. The game in Problem 4.

when she patronizes a seller with a low price.) Each buyer's preferences are represented by the expected value of a payoff function that assigns the payoff 0 to not trading and the payoff $1 - p$ to purchasing one unit of the good at the price p. (Neither buyer values more than one unit.)

(a) Formulate the strategic game that models this situation.

(b) For any pair (p_1, p_2) of prices with $0 < p_i < 1$ for $i = 1, 2$ and $2p_1 - 1 < p_2 < \frac{1}{2}(1 + p_1)$, find the Nash equilibria of the game. (If you claim the game has a mixed strategy equilibrium, be sure to check that the probabilities assigned to each action are nonnegative and at most 1.)

4. Members of a single population of car drivers are randomly matched in pairs when they simultaneously approach intersections from different directions. In each interaction, each driver can either stop or continue. The drivers' preferences are represented by the expected value of the payoff functions given in Figure 1; the parameter ε, with $0 < \varepsilon < 1$, reflects the fact that each driver dislikes being the only one to stop.

   (a) Find the Nash equilibria (equilibria?) of the game in which all players use the *same* strategy. (Find both the equilibrium strategies and the equilibrium payoffs.)

   (b) Now suppose that drivers are (re)educated to feel guilty about choosing *Continue*. Assume that their payoffs when choosing *Continue* fall by $\delta > 0$, so that the entry $(2, 1 - \varepsilon)$ in Figure 1 is replaced by $(2 - \delta, 1 - \varepsilon)$, the entry $(1 - \varepsilon, 2)$ is replaced by $(1 - \varepsilon, 2 - \delta)$, and the entry $(0, 0)$ is replaced by $(-\delta, -\delta)$. Show that all drivers are *better off* in the symmetric equilibrium of this game than they are in the symmetric equilibrium of the original game. Why is the society better off if everyone feels guilty about being aggressive? (The equilibrium of this game, like that of the game of expert diagnosis, may attractively be interpreted as representing a steady state in which some members of the population always choose one action and other members always choose the other action.)