ECO316: Applied game theory

Lecture 5

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Application: expert diagnosis

- Market contains consumers and experts
- Every consumer has a problem (computer broken, car rattling, furnace sputtering, tooth hurts, . . .)
- Consumers unable to diagnose problem
- Experts able to diagnose problem
- But expert does not have to report correct diagnosis
- Depending on diagnosis, consumer may or may not hire expert
  - May put up with problem or fix it themselves
- What fraction of experts will report honestly? What fraction of consumers will hire experts? Could regulation improve outcome?

Application: expert diagnosis

Model

- Each consumer's problem is *major* or *minor*
- Fraction of major problems: r
- Every expert knows whether any given problem is major or minor
- Consumers know only r
- Two possible repairs: *major* and *minor*
  - Major repair fixes both major and minor problem
  - Minor repair fixes only minor problem
- Each consumer decides whether to hire expert after hearing diagnosis
- Consumer who doesn't hire expert fixes it herself or puts up with problem

Application: expert diagnosis

Payoffs

Experts For major problem, sell and perform major repair: π^*
For minor problem,
sell minor repair: $\pi \leq \pi^*$
sell *major* repair: $\pi' > \pi$

Consumers Major repair by expert costs E
Fixing major problem herself costs $E' > E$
Minor repair by expert costs $I < E$
Fixing minor problem herself costs $I' > I$

Assume $I' < E$ (fixing minor problem yourself is cheaper than having expert do major repair)

Application: expert diagnosis

Assumptions

- consumer always hires expert who recommends minor repair
  - I is smallest cost consumer can possibly pay
- expert always recommends major repair for major problem
  - minor repair does not fix major problem

Application: expert diagnosis

Strategic game

Players Expert and consumer

Actions Expert: *Honest* (diagnose minor problem as minor), *Dishonest* (diagnose minor problem as major)
Consumer: *Accept* (hire expert whatever their diagnosis), *Reject* (don't hire expert who diagnoses major problem)

Payoffs

Consumer

Expert	*Accept*	*Reject*
Honest	$r\pi^* + (1-r)\pi, -rE - (1-r)I$	$(1-r)\pi, -rE' - (1-r)I$
Dishonest	$r\pi^* + (1-r)\pi', -E$	$0, -rE' - (1-r)I'$

Application: expert diagnosis

Nash equilibria

		Consumer: *Accept*	Consumer: *Reject*
Expert	*Honest*	$r\pi^* + (1-r)\pi\,,\ -rE - (1-r)I$	$(1-r)\pi\,,\ -rE' - (1-r)I$
	Dishonest	$r\pi^* + (1-r)\pi'\,,\ -E$	$0\,,\ -rE' - (1-r)I'$

Expert's best responses:

- Consumer chooses *Accept* $\Rightarrow$ *Dishonest* $\succ$ *Honest*
- Consumer chooses *Reject* $\Rightarrow$ *Honest* $\succ$ *Dishonest*

Consumer's best responses:

- Expert chooses *Honest* $\Rightarrow$ *Accept* $\succ$ *Reject*
- Expert chooses *Dishonest* $\Rightarrow$
  - if $E < rE' + (1-r)I'$ then *Accept* $\succ$ *Reject*
    $\Rightarrow$ pure strategy Nash equilibrium (*Dishonest*, *Accept*)
  - if $E > rE' + (1-r)I'$ then *Reject* $\succ$ *Accept*
    $\Rightarrow$ no pure strategy equilibrium

Application: expert diagnosis

Cost of major repair $>$ expected cost of self-repair

Mixed strategy equilibrium when $E > rE' + (1-r)I'$

Consumer

Expert		Accept (q)	Reject ($1-q$)
	Honest (p)	$r\pi^* + (1-r)\pi\,,\ -rE-(1-r)I$	$(1-r)\pi\,,\ -rE'-(1-r)I$
	Dishonest ($1-p$)	$r\pi^* + (1-r)\pi'\,,\ -E$	$0\,,\ -rE'-(1-r)I'$

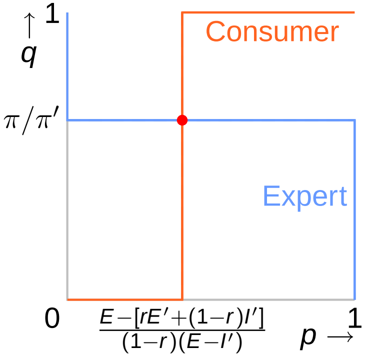
Expert: *Honest* $\succ$ *Dishonest*

$\Rightarrow$

$$q(r\pi^*+(1-r)\pi)+(1-q)(1-r)\pi > q(r\pi^*+(1-r)\pi')$$

$\Rightarrow$

$$q < \pi/\pi'$$



Application: expert diagnosis

Cost of major repair $>$ expected cost of self-repair

Mixed strategy equilibrium when $E > rE' + (1-r)I'$

Consumer

Expert	Accept (q)	Reject ($1-q$)
Honest (p)	$r\pi^* + (1-r)\pi\,,\ -rE - (1-r)I$	$(1-r)\pi\,,\ -rE' - (1-r)I$
Dishonest ($1-p$)	$r\pi^* + (1-r)\pi'\,,\ -E$	$0\,,\ -rE' - (1-r)I'$

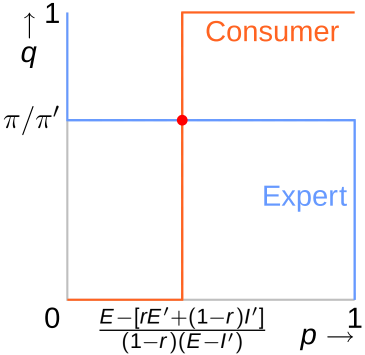
Consumer: *Accept* $\succ$ *Reject*

$\Rightarrow$

$$p(-rE-(1-r)I)+(1-p)(-E) > p(-rE'-(1-r)I) + (1-p)(-rE'-(1-r)I')$$

$\Rightarrow$

$$p > \frac{E - [rE' + (1-r)I']}{(1-r)(E-I')}$$



Application: expert diagnosis

Cost of major repair $>$ expected cost of self-repair

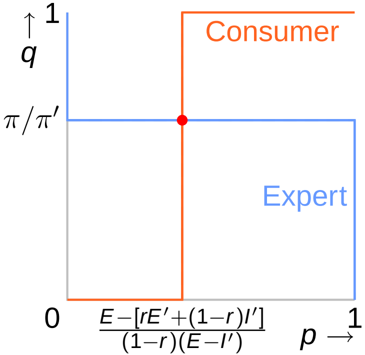
Mixed strategy equilibrium when $E > rE' + (1-r)I'$

		Consumer	
		Accept (q)	*Reject* ($1-q$)
Exp	*Honest* (p)	$r\pi^* + (1-r)\pi\,,\ -rE-(1-r)I$	$(1-r)\pi\,,\ -rE'-(1-r)I$
	Dishonest ($1-p$)	$r\pi^* + (1-r)\pi'\,,\ -E$	$0\,,\ -rE'-(1-r)I'$

Unique Nash equilibrium with

$$p = \frac{E - [rE' + (1-r)I']}{(1-r)(E-I')}$$

$$q = \pi/\pi'$$



Application: expert diagnosis

Mixed strategy equilibrium when $E > rE' + (1-r)I'$

$$p = \frac{E - [rE' + (1-r)I']}{(1-r)(E-I')}$$
$$q = \pi/\pi'$$

We have $p > 0$ and $0 < q < 1$. Also

$$p = \frac{E - [rE' + (1-r)I']}{(1-r)(E-I')} = 1 - \frac{r(E'-E)}{(1-r)(E-I')}$$

so $p < 1$.

Hence equilibrium in which

- some experts are honest, some dishonest
- some consumers accept major diagnoses (“credulous”), some reject them (“wary”)

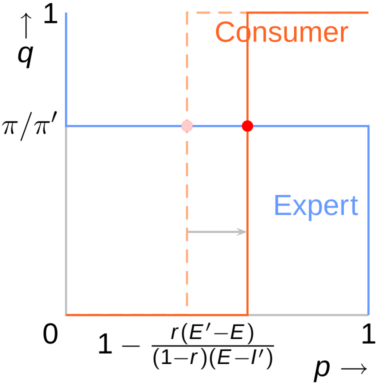
Application: expert diagnosis

Mixed strategy equilibrium: comparative statics

$$\text{prob. expert honest} = p = 1 - \frac{r(E' - E)}{(1-r)(E - I')}$$

$$\text{prob. consumer accepts major diagnosis} = q = \pi/\pi'$$

- Major problems less common (more reliable cars) $\Rightarrow r \downarrow$
  - $\Rightarrow$ $p \uparrow$, q unchanged
  - $\Rightarrow$ more experts honest, consumer behavior unchanged
  - intuition: major problems less common $\Rightarrow$ consumer has less to lose from ignoring expert's advice, so probability of expert being honest must rise for her advice to be heeded



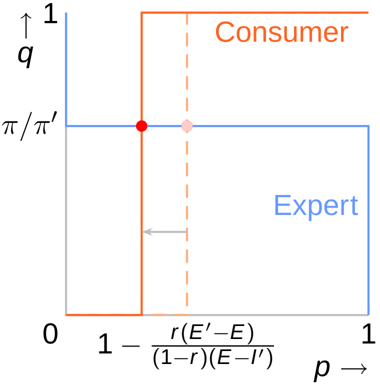
Application: expert diagnosis

Mixed strategy equilibrium: comparative statics

$$\text{prob. expert honest} = p = 1 - \frac{r(E' - E)}{(1 - r)(E - I')}$$

$$\text{prob. consumer accepts major diagnosis} = q = \pi/\pi'$$

- Major repairs less expensive relative to minor ones (technical advance?) $\Rightarrow E \downarrow$
  - $\Rightarrow$ $p \downarrow$, q unchanged
  - $\Rightarrow$ fewer experts honest, consumer behavior unchanged
  - intuition: major repairs less costly $\Rightarrow$ consumer has more to lose from ignoring expert's advice, so she heeds the advice even if experts are less likely to be honest



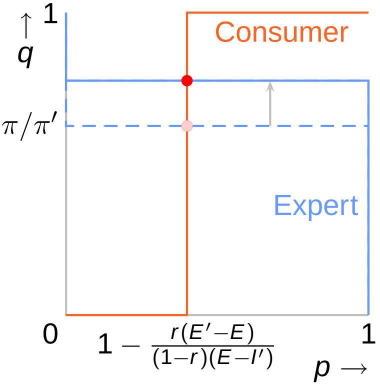
Application: expert diagnosis

Mixed strategy equilibrium: comparative statics

$$\text{prob. expert honest} = p = 1 - \frac{r(E' - E)}{(1-r)(E - I')}$$

$$\text{prob. consumer accepts major diagnosis} = q = \pi/\pi'$$

- $\pi' \downarrow$ (better regulation, so that fraud is harder): q increases
  - $\Rightarrow$ $q \uparrow$, p unchanged
  - $\Rightarrow$ consumers are less wary—they are more likely to accept diagnoses
  - intuition: experts have less to gain from being dishonest, so it pays for them to be dishonest only if consumers are less wary (note: fraud unchanged!)

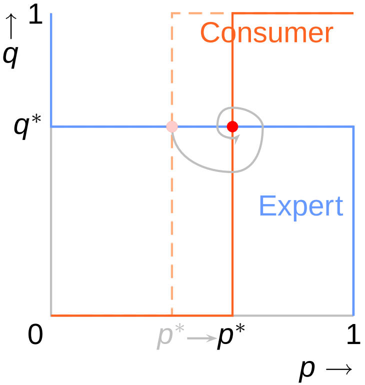


Application: expert diagnosis

Mixed strategy equilibrium: possible dynamics

- Start at equilibrium
- Parameter changes $\Rightarrow$ how is new equilibrium reached?

- $r \downarrow \Rightarrow$ consumer's best response function shifts right
- Given old p^*, best q is now 0, so q starts decreasing
- When q decreases, best p is 1, so p starts increasing
- As long as $p <$ new p^*, best q is 0, so q decreases

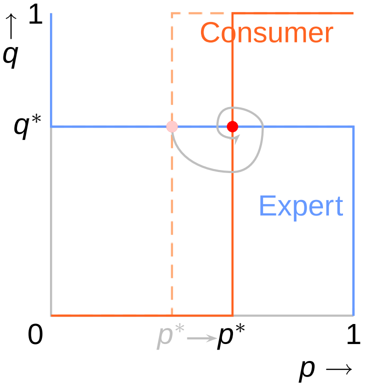


Application: expert diagnosis

Mixed strategy equilibrium: possible dynamics

- Start at equilibrium
- Parameter changes $\Rightarrow$ how is new equilibrium reached?

- Once $p >$ new p^*, best q is 1, so q increases
- When q increases above q^*, best p is zero, so p decreases
- Depending on adjustment speeds, new equilibrium may eventually be reached



Application: expert diagnosis

Nash equilibrium: summary

- Price of major repair less than expected cost of consumer fixing problem themselves $\Rightarrow$ all experts dishonest, all consumers credulous (pure strategy Nash equilibrium)
- Price of major repair greater than expected cost of consumer fixing problem themselves $\Rightarrow$ some experts dishonest, some honest; some consumers credulous, some wary (mixed strategy Nash equilibrium)
- Comparative statics:
  - major problems less common $\Rightarrow$ more experts honest, consumer behavior unaffected
  - major repairs less expensive $\Rightarrow$ fewer experts honest, consumer behavior unaffected
  - less profit from major repair of minor problem $\Rightarrow$ consumers less wary, expert behavior unaffected

Application: reporting a crime (“volunteer’s dilemma”)

- Many people witness a crime
- One person’s reporting crime to police suffices
- When deciding whether to report, each person doesn’t know whether anyone else has reported
- A person who reports bears a cost c
- If the crime is reported, everyone obtains benefit $v > c$
- How many people report? How does number depend on size of group?

Application: reporting a crime ("volunteer's dilemma")

Strategic game

Players n individuals

Actions For each player, $\{Call, Don't\ call\}$

Payoffs For each player i,

$$u_i(a) = \begin{cases} v - c & \text{if } a_i = Call \\ v & \text{if } a_i = Don't\ call \text{ and} \\ & \quad a_j = Call \text{ for some } j \neq i \\ 0 & \text{if } a_j = Don't\ call \text{ for all } j \end{cases}$$

Application: reporting a crime (“volunteer’s dilemma”)

Nash equilibria

- Equilibria in pure strategies?
  - No player calls? Not NE
  - Every player calls? Not NE
  - So no *symmetric* NE
  - n pure NEs, in each of which exactly one player calls
  - How can these equilibria be realized? For an equilibrium in which player 1 calls, who is player 1?
- Look for *symmetric* equilibrium in *mixed* strategies

Application: reporting a crime ("volunteer's dilemma")

Mixed strategy Nash equilibrium

In mixed strategy equilibrium in which every player calls with same probability p with $0 < p < 1$,

$$\text{payoff if player calls} = \text{payoff if player doesn't call}$$

$\Rightarrow$

$$v - c = 0 \cdot \Pr\{\text{no one else calls}\} + v \cdot \Pr\{\geq \text{one other person calls}\}$$

$\Rightarrow$

$$v - c = v \cdot (1 - \Pr\{\text{no one else calls}\}),$$

$\Rightarrow$

$$c/v = \Pr\{\text{No one else calls}\} = (1 - p)^{n-1}$$

$\Rightarrow$

$$p = 1 - (c/v)^{1/(n-1)}$$

Application: reporting a crime ("volunteer's dilemma")

Mixed strategy Nash equilibrium

Conclusion: game has a symmetric mixed strategy Nash equilibrium, in which every player calls with probability

$$p = 1 - (c/v)^{1/(n-1)}$$

(Note: this number is between 0 and 1.)

Application: reporting a crime ("volunteer's dilemma")

Mixed strategy Nash equilibrium: comparative statics

$$p = 1 - (c/v)^{1/(n-1)}$$

- $n \uparrow \Rightarrow p \downarrow$: more people $\Rightarrow$ each is less likely to call
- Probability that at least one person calls:

$$\begin{aligned}
\Pr\{\text{at least one person calls}\} \\
&= 1 - \Pr\{\text{no one calls}\} \\
&= 1 - \Pr\{i \text{ does not call}\} \Pr\{\text{no one else calls}\} \\
&= 1 - (1-p)(c/v)
\end{aligned}$$

Because $n \uparrow \Rightarrow p \downarrow$,

$$n \uparrow \Rightarrow \Pr\{\text{at least one person calls}\} \downarrow$$

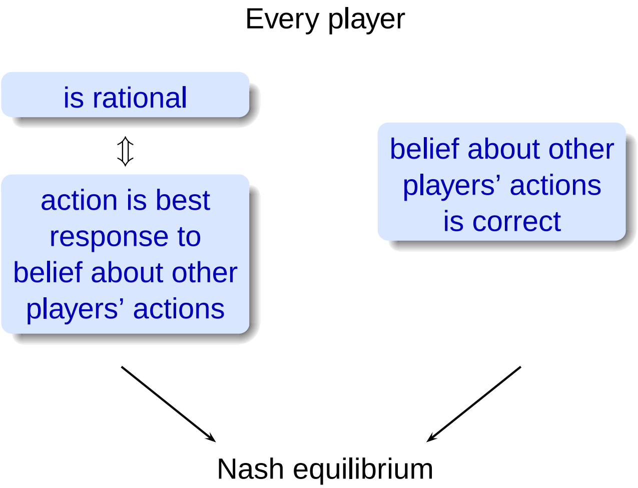
$\Rightarrow$ the more people, the *less* likely the police are informed!

Application: reporting a crime (“volunteer’s dilemma”)

Summary

- n asymmetric pure strategy Nash equilibria—hard to see how they could occur
- Unique symmetric mixed strategy Nash equilibrium, in which each person calls with positive probability less than 1
- $n \uparrow \Rightarrow$ each person is less likely to call (not surprising)
- $n \uparrow \Rightarrow$ probability that *at least one person* calls is less likely
    - More generally, in a large group a collectively beneficial action is less likely to be taken than in a small one
    - For example, result suggests that a broken streetlight is less likely to be reported if it is outside an apartment block than if it is in an area of low-density housing

Rationality and equilibrium



Rationality and equilibrium

Every player

is rational

⇕

action is best response to belief about other players' actions

believes that other players are rational

. . . and believes that other players believe she is rational

. . . and believes that other players believe she believes they are rational

. . . and so on

?