ECO316: Applied game theory

Lecture 5

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2017.10.5

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Table of contents

Application: expert diagnosis

- Market contains consumers and experts

Application: expert diagnosis

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- Every consumer has a problem (computer broken, car rattling, furnace sputtering, tooth hurts, . . .)

Application: expert diagnosis

- Market contains consumers and experts
- Every consumer has a problem (computer broken, car rattling, furnace sputtering, tooth hurts, . . .)
- Consumers unable to diagnose problem

Application: expert diagnosis

- Market contains consumers and experts
- Every consumer has a problem (computer broken, car rattling, furnace sputtering, tooth hurts, . . .)
- Consumers unable to diagnose problem
- Experts able to diagnose problem

Application: expert diagnosis

- Market contains consumers and experts
- Every consumer has a problem (computer broken, car rattling, furnace sputtering, tooth hurts, . . .)
- Consumers unable to diagnose problem
- Experts able to diagnose problem
- But expert does not have to report correct diagnosis

Application: expert diagnosis

- Market contains consumers and experts
- Every consumer has a problem (computer broken, car rattling, furnace sputtering, tooth hurts, . . .)
- Consumers unable to diagnose problem
- Experts able to diagnose problem
- But expert does not have to report correct diagnosis
- Depending on diagnosis, consumer may or may not hire expert

Application: expert diagnosis

- Market contains consumers and experts
- Every consumer has a problem (computer broken, car rattling, furnace sputtering, tooth hurts, . . .)
- Consumers unable to diagnose problem
- Experts able to diagnose problem
- But expert does not have to report correct diagnosis
- Depending on diagnosis, consumer may or may not hire expert
  - May put up with problem or fix it themselves

Application: expert diagnosis

- Market contains consumers and experts
- Every consumer has a problem (computer broken, car rattling, furnace sputtering, tooth hurts, . . .)
- Consumers unable to diagnose problem
- Experts able to diagnose problem
- But expert does not have to report correct diagnosis
- Depending on diagnosis, consumer may or may not hire expert
    - May put up with problem or fix it themselves
- What fraction of experts will report honestly? What fraction of consumers will hire experts? Could regulation improve outcome?

Application: expert diagnosis

Model

- Each consumer's problem is *major* or *minor*

Application: expert diagnosis

Model

- Each consumer's problem is *major* or *minor*
- Fraction of major problems: r

Application: expert diagnosis

Model

- Each consumer's problem is *major* or *minor*
- Fraction of major problems: r
- Every expert knows whether any given problem is major or minor

Application: expert diagnosis

Model

- Each consumer's problem is *major* or *minor*
- Fraction of major problems: r
- Every expert knows whether any given problem is major or minor
- Consumers know only r

Application: expert diagnosis

Model

- Each consumer's problem is *major* or *minor*
- Fraction of major problems: r
- Every expert knows whether any given problem is major or minor
- Consumers know only r
- Two possible repairs: *major* and *minor*

Application: expert diagnosis

Model

- Each consumer's problem is *major* or *minor*
- Fraction of major problems: r
- Every expert knows whether any given problem is major or minor
- Consumers know only r
- Two possible repairs: *major* and *minor*
    - Major repair fixes both major and minor problem

Application: expert diagnosis

Model

- Each consumer's problem is *major* or *minor*
- Fraction of major problems: r
- Every expert knows whether any given problem is major or minor
- Consumers know only r
- Two possible repairs: *major* and *minor*
  - Major repair fixes both major and minor problem
  - Minor repair fixes only minor problem

Application: expert diagnosis

Model

- Each consumer's problem is *major* or *minor*
- Fraction of major problems: r
- Every expert knows whether any given problem is major or minor
- Consumers know only r
- Two possible repairs: *major* and *minor*
  - Major repair fixes both major and minor problem
  - Minor repair fixes only minor problem
- Each consumer decides whether to hire expert after hearing diagnosis

Application: expert diagnosis

Model

- Each consumer's problem is *major* or *minor*
- Fraction of major problems: r
- Every expert knows whether any given problem is major or minor
- Consumers know only r
- Two possible repairs: *major* and *minor*
  - Major repair fixes both major and minor problem
  - Minor repair fixes only minor problem
- Each consumer decides whether to hire expert after hearing diagnosis
- Consumer who doesn't hire expert fixes it herself or puts up with problem

Application: expert diagnosis

Payoffs

Experts For major problem, sell and perform major repair: π^*

Application: expert diagnosis

Payoffs

Experts For major problem, sell and perform major repair: π^*
For minor problem,
sell minor repair: π

Application: expert diagnosis

Payoffs

Experts For major problem, sell and perform major repair: π^*
For minor problem,
sell minor repair: $\pi \leq \pi^*$

Application: expert diagnosis

Payoffs

Experts For major problem, sell and perform major repair: π^*
For minor problem,
sell minor repair: $\pi \leq \pi^*$
sell *major* repair: $\pi' > \pi$

Application: expert diagnosis

Payoffs

Experts For major problem, sell and perform major repair: π^*
For minor problem,
sell minor repair: $\pi \leq \pi^*$
sell *major* repair: $\pi' > \pi$

Expert might charge for major repair but do minor one (car mechanic charges for new transmission but only tightens bolt)
or might do unnecessary major repair ($\pi' = \pi^* > \pi$) (dentist does root canal when filling would suffice)

Application: expert diagnosis

Payoffs

Experts For major problem, sell and perform major repair: π^*
For minor problem,
sell minor repair: $\pi \leq \pi^*$
sell *major* repair: $\pi' > \pi$

Consumers Major repair by expert costs E

Application: expert diagnosis

Payoffs

Experts For major problem, sell and perform major repair: π^*
For minor problem,
sell minor repair: $\pi \leq \pi^*$
sell *major* repair: $\pi' > \pi$

Consumers Major repair by expert costs E
Fixing major problem herself costs $E' > E$

Application: expert diagnosis

Payoffs

Experts For major problem, sell and perform major repair: π^*
For minor problem,
sell minor repair: $\pi \leq \pi^*$
sell *major* repair: $\pi' > \pi$

Consumers Major repair by expert costs E
Fixing major problem herself costs $E' > E$
Minor repair by expert costs $I < E$

Application: expert diagnosis

Payoffs

Experts For major problem, sell and perform major repair: π^*
For minor problem,
sell minor repair: $\pi \leq \pi^*$
sell *major* repair: $\pi' > \pi$

Consumers Major repair by expert costs E
Fixing major problem herself costs $E' > E$
Minor repair by expert costs $I < E$
Fixing minor problem herself costs $I' > I$

Application: expert diagnosis

Payoffs

Experts
For major problem, sell and perform major repair: π^*
For minor problem,
sell minor repair: $\pi \leq \pi^*$
sell *major* repair: $\pi' > \pi$

Consumers
Major repair by expert costs E
Fixing major problem herself costs $E' > E$
Minor repair by expert costs $I < E$
Fixing minor problem herself costs $I' > I$

Assume $I' < E$ (fixing minor problem yourself is cheaper than having expert do major repair)

Application: expert diagnosis

Assumptions

- consumer always hires expert who recommends minor repair
  - I is smallest cost consumer can possibly pay

Application: expert diagnosis

Assumptions

- consumer always hires expert who recommends minor repair
  - I is smallest cost consumer can possibly pay
- expert always recommends major repair for major problem
  - minor repair does not fix major problem

Application: expert diagnosis

Strategic game

Players Expert and consumer

Application: expert diagnosis

Strategic game

Players Expert and consumer

Actions Expert: *Honest* (diagnose minor problem as minor), *Dishonest* (diagnose minor problem as major)

Application: expert diagnosis

Reminder: expert always diagnoses *major* problem as *major*

Strategic game

Players Expert and consumer

Actions Expert: *Honest* (diagnose minor problem as minor), *Dishonest* (diagnose minor problem as major)

Application: expert diagnosis

Strategic game

Players	Expert and consumer
Actions	Expert: *Honest* (diagnose minor problem as minor), *Dishonest* (diagnose minor problem as major) Consumer: *Accept* (hire expert whatever their diagnosis), *Reject* (don't hire expert who diagnoses major problem)

Application: expert diagnosis

Strategic game

Players Expert and consumer

Actions Expert: *Honest* (diagnos minor), *Dishonest* (diagn major)

Consumer: *Accept* (hire expert whatever their diagnosis), *Reject* (don't hire expert who diagnoses major problem)

Reminder: consumer always accepts *minor* diagnosis

Application: expert diagnosis

Strategic game

Players Expert and consumer

Actions Expert: *Honest* (diagnose minor problem as minor), *Dishonest* (diagnose minor problem as major)
Consumer: *Accept* (hire expert whatever their diagnosis), *Reject* (don't hire expert who diagnoses major problem)

Payoffs

		Consumer: *Accept*	Consumer: *Reject*
Expert	*Honest*		
	Dishonest		

Application: expert diagnosis

Strategic game

Players Expert and consumer

Actions Expert: *Honest* (diagnose minor problem as minor), *Dishonest* (diagnose minor problem as major)
Consumer: *Accept* (hire expert whatever their diagnosis), *Reject* (don't hire expert who diagnoses major problem)

Payoffs

sells to all consumers

		Consumer	
		Accept	*Reject*
Expert	*Honest*		
	Dishonest		

Application: expert diagnosis

Strategic game

Players Expert and consumer

Actions Expert: *Honest* (diagnose minor problem as minor), *Dishonest* (diagnose minor problem as major)
Consumer: *Accept* (hire expert whatever their diagnosis), *Reject* (don't hire expert who diagnoses major problem)

Payoffs

sells to all consumers

		Consumer	
		Accept	*Reject*
Expert	*Honest*	$r\pi^* + (1-r)\pi,$	
	Dishonest		

Application: expert diagnosis

Strategic game

Players Expert and consumer

Actions Expert: *Honest* (diagnose minor problem as minor), *Dishonest* (diagnose minor problem as major)
Consumer: *Accept* (hire expert whatever their diagnosis), *Reject* (don't hire expert who diagnoses major problem)

Payoffs

sells only to consumers with minor problems

		Accept	*Reject*
Expert	*Honest*	$r\pi^* + (1-r)\pi,$	
	Dishonest		

Application: expert diagnosis

Strategic game

Players Expert and consumer

Actions Expert: *Honest* (diagnose minor problem as minor), *Dishonest* (diagnose minor problem as major)
Consumer: *Accept* (hire expert whatever their diagnosis), *Reject* (don't hire expert who diagnoses major problem)

Payoffs

sells only to consumers with minor problems

		Accept	*Reject*
Expert	*Honest*	$r\pi^* + (1-r)\pi,$	$(1-r)\pi,$
	Dishonest		

Application: expert diagnosis

Strategic game

Players Expert and consumer

Actions Expert: *Honest* (diagnose minor problem as minor), *Dishonest* (diagnose minor problem as major)
Consumer: *Accept* (hire expert whatever their diagnosis), *Reject* (don't hire expert who diagnoses major problem)

Payoffs

Consumer

		Accept	*Reject*
Expert	*Honest*	$r\pi^* + (1-r)\pi,$	$(1-r)\pi,$
	Dishonest		

sells to all consumers and makes π^* on major repairs, π' on minor ones

Application: expert diagnosis

Strategic game

Players Expert and consumer

Actions Expert: *Honest* (diagnose minor problem as minor), *Dishonest* (diagnose minor problem as major)
Consumer: *Accept* (hire expert whatever their diagnosis), *Reject* (don't hire expert who diagnoses major problem)

Payoffs

Consumer

Expert		*Accept*	*Reject*
	Honest	$r\pi^* + (1-r)\pi$,	$(1-r)\pi$,
	Dishonest	$r\pi^* + (1-r)\pi'$,	

sells to all consumers and makes π^* on major repairs, π' on minor ones

Application: expert diagnosis

Strategic game

Players Expert and consumer

Actions Expert: *Honest* (diagnose minor problem as minor), *Dishonest* (diagnose minor problem as major)
Consumer: *Accept* (hire expert whatever their diagnosis), *Reject* (don't hire expert who diagnoses major problem)

Payoffs

Consumer

		Accept	*Reject*
Expert	*Honest*	$r\pi^* + (1-r)\pi,$	$(1-r)\pi,$
	Dishonest	$r\pi^* + (1-r)\pi',$	

sells to no consumers

Application: expert diagnosis

Strategic game

Players Expert and consumer

Actions Expert: *Honest* (diagnose minor problem as minor), *Dishonest* (diagnose minor problem as major)
Consumer: *Accept* (hire expert whatever their diagnosis), *Reject* (don't hire expert who diagnoses major problem)

Payoffs

Consumer

Expert	*Accept*	*Reject*
Honest	$r\pi^* + (1-r)\pi,$	$(1-r)\pi,$
Dishonest	$r\pi^* + (1-r)\pi',$	$0,$

sells to no consumers

Application: expert diagnosis

Strategic game

Players Expert and consumer

Actions Expert: *Honest* (diagnose minor problem as minor), *Dishonest* (diagnose minor problem as major)
Consumer: *Accept* (hire expert whatever their diagnosis), *Reject* (don't hire expert who diagnoses major problem)

Payoffs

pays E if problem is major, I if problem is minor

umer

		Accept	*Reject*
Expert	*Honest*	$r\pi^* + (1-r)\pi,$	$(1-r)\pi,$
	Dishonest	$r\pi^* + (1-r)\pi',$	$0,$

Application: expert diagnosis

Strategic game

Players Expert and consumer

Actions Expert: *Honest* (diagnose minor problem as minor), *Dishonest* (diagnose minor problem as major)
Consumer: *Accept* (hire expert whatever their diagnosis), *Reject* (don't hire expert who diagnoses major problem)

Payoffs

pays E if problem is major,
I if problem is minor

umer

		Accept	Reject
Expert	*Honest*	$r\pi^* + (1-r)\pi, -rE - (1-r)I$	$(1-r)\pi,$
	Dishonest	$r\pi^* + (1-r)\pi',$	$0,$

Application: expert diagnosis

Strategic game

Players Expert and consumer

Actions Expert: *Honest* (diagnose minor problem as minor), *Dishonest* (diagnose minor problem as major)
Consumer: *Accept* (hire expert whatever their diagnosis), *Reject* (don't hire expert who diagnoses major problem)

Payoffs

Cor

fixes major problem herself, pays I if problem is minor

		Consumer: *Accept*	Consumer: *Reject*
Expert	*Honest*	$r\pi^* + (1-r)\pi, -rE - (1-r)I$	$(1-r)\pi,$
	Dishonest	$r\pi^* + (1-r)\pi',$	$0,$

Application: expert diagnosis

Strategic game

Players Expert and consumer

Actions Expert: *Honest* (diagnose minor problem as minor), *Dishonest* (diagnose minor problem as major)
Consumer: *Accept* (hire expert whatever their diagnosis), *Reject* (don't hire expert who diagnoses major problem)

Payoffs

fixes major problem herself, pays I if problem is minor

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		Accept	*Reject*
Expert	*Honest*	$r\pi^* + (1-r)\pi, -rE - (1-r)I$	$(1-r)\pi, -rE' - (1-r)I$
	Dishonest	$r\pi^* + (1-r)\pi',$	$0,$

Application: expert diagnosis

Strategic game

Players Expert and consumer

Actions Expert: *Honest* (diagnose minor problem as minor), *Dishonest* (diagnose minor problem as major)
Consumer: *Accept* (hire expert whatever their diagnosis), *Reject* (don't hire expert who diagnoses major problem)

Payoffs

Consumer

Expert		*Accept*	*Reject*
	Honest	$r\pi^* + (1-r)\pi, -rE - (1-r)I$	$(1-r)\pi, -rE' - (1-r)I$
	Dishonest	$r\pi^* + (1-r)\pi',$	$0,$

pays E for all problems

Application: expert diagnosis

Strategic game

Players Expert and consumer

Actions Expert: *Honest* (diagnose minor problem as minor), *Dishonest* (diagnose minor problem as major)
Consumer: *Accept* (hire expert whatever their diagnosis), *Reject* (don't hire expert who diagnoses major problem)

Payoffs

Consumer

Expert		*Accept*	*Reject*
	Honest	$r\pi^* + (1-r)\pi, -rE - (1-r)I$	$(1-r)\pi, -rE' - (1-r)I$
	Dishonest	$r\pi^* + (1-r)\pi', -E$	$0,$

pays E for all problems

Application: expert diagnosis

Strategic game

Players Expert and consumer

Actions Expert: *Honest* (diagnose minor problem as minor), *Dishonest* (diagnose minor problem as major)
Consumer: *Accept* (hire expert whatever their diagnosis), *Reject* (don't hire expert who diagnoses major problem)

Payoffs

Consumer

Expert		*Accept*	*Reject*
	Honest	$r\pi^* + (1-r)\pi, -rE - (1-r)I$	$(1-r)\pi, -rE' - (1-r)I$
	Dishonest	$r\pi^* + (1-r)\pi', -E$	$0,$

fixes all problems herself

Application: expert diagnosis

Strategic game

Players Expert and consumer

Actions Expert: *Honest* (diagnose minor problem as minor), *Dishonest* (diagnose minor problem as major)
Consumer: *Accept* (hire expert whatever their diagnosis), *Reject* (don't hire expert who diagnoses major problem)

Payoffs

Consumer

Expert		*Accept*	*Reject*
	Honest	$r\pi^* + (1-r)\pi, -rE - (1-r)I$	$(1-r)\pi, -rE' - (1-r)I$
	Dishonest	$r\pi^* + (1-r)\pi', -E$	$0, -rE' - (1-r)I'$

fixes all problems herself

Application: expert diagnosis

Nash equilibria

		Consumer	
		Accept	*Reject*
Expert	*Honest*	$r\pi^* + (1-r)\pi\,,\ -rE - (1-r)I$	$(1-r)\pi\,,\ -rE' - (1-r)I$
	Dishonest	$r\pi^* + (1-r)\pi'\,,\ -E$	$0\,,\ -rE' - (1-r)I'$

Application: expert diagnosis

Nash equilibria

		Consumer: *Accept*	Consumer: *Reject*
Expert	*Honest*	$r\pi^* + (1-r)\pi\,,\ -rE-(1-r)I$	$(1-r)\pi\,,\ -rE'-(1-r)I$
	Dishonest	$r\pi^* + (1-r)\pi'\,,\ -E$	$0\,,\ -rE'-(1-r)I'$

Expert's best responses:

- Consumer chooses *Accept* $\Rightarrow$

Application: expert diagnosis

Nash equilibria

		Consumer	
		Accept	*Reject*
Expert	*Honest*	$r\pi^* + (1-r)\pi\,,\ -rE - (1-r)I$	$(1-r)\pi\,,\ -rE' - (1-r)I$
	Dishonest	$r\pi^* + (1-r)\pi'\,,\ -E$	$0\,,\ -rE' - (1-r)I'$

Expert's best responses:

- Consumer chooses *Accept* $\Rightarrow$ *Dishonest* $\succ$ *Honest*

Application: expert diagnosis

Nash equilibria

		Consumer: *Accept*	Consumer: *Reject*
Expert	*Honest*	$r\pi^* + (1-r)\pi, -rE - (1-r)I$	$(1-r)\pi, -rE' - (1-r)I$
	Dishonest	$r\pi^* + (1-r)\pi', -E$	$0, -rE' - (1-r)I'$

Expert's best responses:

- Consumer chooses *Accept* $\Rightarrow$ *Dishonest* $\succ$ *Honest*
- Consumer chooses *Reject* $\Rightarrow$

Application: expert diagnosis

Nash equilibria

		Consumer: *Accept*	Consumer: *Reject*
Expert	*Honest*	$r\pi^* + (1-r)\pi\,,\ -rE - (1-r)I$	$(1-r)\pi\,,\ -rE' - (1-r)I$
	Dishonest	$r\pi^* + (1-r)\pi'\,,\ -E$	$0\,,\ -rE' - (1-r)I'$

Expert's best responses:

- Consumer chooses *Accept* $\Rightarrow$ *Dishonest* $\succ$ *Honest*
- Consumer chooses *Reject* $\Rightarrow$ *Honest* $\succ$ *Dishonest*

Application: expert diagnosis

Nash equilibria

		Consumer: *Accept*	Consumer: *Reject*
Expert	*Honest*	$r\pi^* + (1-r)\pi\,,\ -rE - (1-r)I$	$(1-r)\pi,\ -rE' - (1-r)I$
	Dishonest	$r\pi^* + (1-r)\pi',\ -E$	$0\,,\ -rE' - (1-r)I'$

Expert's best responses:

- Consumer chooses *Accept* $\Rightarrow$ *Dishonest* $\succ$ *Honest*
- Consumer chooses *Reject* $\Rightarrow$ *Honest* $\succ$ *Dishonest*

Consumer's best responses:

- Expert chooses *Honest* $\Rightarrow$

Application: expert diagnosis

Nash equilibria

		Consumer	
		Accept	*Reject*
Expert	*Honest*	$r\pi^* + (1-r)\pi\,,\ -rE - (1-r)I$	$(1-r)\pi,\ -rE' - (1-r)I$
	Dishonest	$r\pi^* + (1-r)\pi',\ -E$	$0\,,\ -rE' - (1-r)I'$

Expert's best responses:

- Consumer chooses *Accept* $\Rightarrow$ *Dishonest* $\succ$ *Honest*
- Consumer chooses *Reject* $\Rightarrow$ *Honest* $\succ$ *Dishonest*

Consumer's best responses:

- Expert chooses *Honest* $\Rightarrow$ *Accept* $\succ$ *Reject*

Application: expert diagnosis

Nash equilibria

		Consumer	
		Accept	*Reject*
Expert	*Honest*	$r\pi^* + (1-r)\pi\,,\ -rE - (1-r)I$	$(1-r)\pi\,,\ -rE' - (1-r)I$
	Dishonest	$r\pi^* + (1-r)\pi'\,,\ -E$	$0\,,\ -rE' - (1-r)I'$

Expert's best responses:

- Consumer chooses *Accept* $\Rightarrow$ *Dishonest* $\succ$ *Honest*
- Consumer chooses *Reject* $\Rightarrow$ *Honest* $\succ$ *Dishonest*

Consumer's best responses:

- Expert chooses *Honest* $\Rightarrow$ *Accept* $\succ$ *Reject*
- Expert chooses *Dishonest* $\Rightarrow$

Application: expert diagnosis

Nash equilibria

		Consumer	
		Accept	*Reject*
Expert	*Honest*	$r\pi^* + (1-r)\pi\,,\ -rE - (1-r)I$	$(1-r)\pi\,,\ -rE' - (1-r)I$
	Dishonest	$r\pi^* + (1-r)\pi'\,,\ -E$	$0\,,\ -rE' - (1-r)I'$

Expert's best responses:

- Consumer chooses *Accept* $\Rightarrow$ *Dishonest* $\succ$ *Honest*
- Consumer chooses *Reject* $\Rightarrow$ *Honest* $\succ$ *Dishonest*

Consumer's best responses:

- Expert chooses *Honest* $\Rightarrow$ *Accept* $\succ$ *Reject*
- Expert chooses *Dishonest* $\Rightarrow$
  - if $E < rE' + (1-r)I'$ then *Accept* $\succ$ *Reject*

major repair cheaper than
expected cost of self-repair

Application: expert diagnosis

Nash equilibria

		Consumer: *Accept*	Consumer: *Reject*
Expert	*Honest*	$r\pi^* + (1-r)\pi\,,\ -rE - (1-r)I$	$(1-r)\pi,\ -rE' - (1-r)I$
	Dishonest	$r\pi^* + (1-r)\pi',\ -E$	$0\,,\ -rE' - (1-r)I'$

Expert's best responses:

- Consumer chooses *Accept* $\Rightarrow$ *Dishonest* $\succ$ *Honest*
- Consumer chooses *Reject* $\Rightarrow$ *Honest* $\succ$ *Dishonest*

Consumer's best responses:

- Expert chooses *Honest* $\Rightarrow$ *Accept* $\succ$ *Reject*
- Expert chooses *Dishonest* $\Rightarrow$
  - if $E < rE' + (1-r)I'$ then *Accept* $\succ$ *Reject*
    $\Rightarrow$ pure strategy Nash equilibrium $(\textit{Dishonest}, \textit{Accept})$

Application: expert diagnosis

Nash equilibria

		Consumer: *Accept*	Consumer: *Reject*
Expert	*Honest*	$r\pi^* + (1-r)\pi\,,\ -rE - (1-r)I$	$(1-r)\pi,\ -rE' - (1-r)I$
	Dishonest	$r\pi^* + (1-r)\pi',\ -E$	$0\,,\ -rE' - (1-r)I'$

Expert's best responses:

- Consumer chooses *Accept* $\Rightarrow$ *Dishonest* $\succ$ *Honest*
- Consumer chooses *Reject* $\Rightarrow$ *Honest* $\succ$ *Dishonest*

Consumer's best responses:

- Expert chooses *Honest* $\Rightarrow$ *Accept* $\succ$ *Reject*
- Expert chooses *Dishonest* $\Rightarrow$

major repair more expensive
than expected cost of self-repair

ccept $\succ$ *Reject*

$\Rightarrow$ pure st tegy Nash equilibrium (*Dishonest*, *Accept*)

  - if $E > rE' + (1-r)I'$ then *Reject* $\succ$ *Accept*

Application: expert diagnosis

Nash equilibria

		Consumer	
		Accept	*Reject*
Expert	*Honest*	$r\pi^* + (1-r)\pi\,,\ -rE - (1-r)I$	$(1-r)\pi\,,\ -rE' - (1-r)I$
	Dishonest	$r\pi^* + (1-r)\pi'\,,\ -E$	$0\,,\ -rE' - (1-r)I'$

Expert's best responses:

- Consumer chooses *Accept* $\Rightarrow$ *Dishonest* $\succ$ *Honest*
- Consumer chooses *Reject* $\Rightarrow$ *Honest* $\succ$ *Dishonest*

Consumer's best responses:

- Expert chooses *Honest* $\Rightarrow$ *Accept* $\succ$ *Reject*
- Expert chooses *Dishonest* $\Rightarrow$
  - if $E < rE' + (1-r)I'$ then *Accept* $\succ$ *Reject*
    $\Rightarrow$ pure strategy Nash equilibrium $(\textit{Dishonest}, \textit{Accept})$
  - if $E > rE' + (1-r)I'$ then *Reject* $\succ$ *Accept*
    $\Rightarrow$ no pure strategy equilibrium

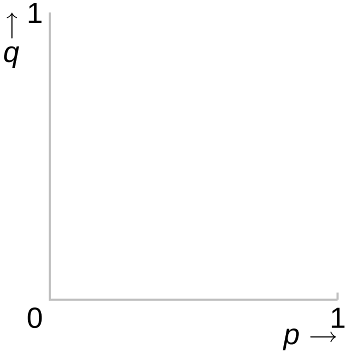
Application: expert diagnosis

Cost of major repair $>$ expected cost of self-repair

Mixed strategy equilibrium when $E > rE' + (1-r)I'$

Consumer

Exp	Accept (q)	Reject $(1-q)$
Honest (p)	$r\pi^* + (1-r)\pi\,,\ -rE - (1-r)I$	$(1-r)\pi\,,\ -rE' - (1-r)I$
Dishonest $(1-p)$	$r\pi^* + (1-r)\pi'\,,\ -E$	$0\,,\ -rE' - (1-r)I'$



Application: expert diagnosis

Cost of major repair $>$ expected cost of self-repair

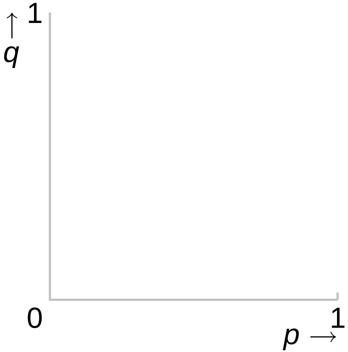
Mixed strategy equilibrium when $E > rE' + (1-r)I'$

Consumer

Exp	Accept (q)	Reject $(1-q)$
Honest (p)	$r\pi^* + (1-r)\pi$, $-rE - (1-r)I$	$(1-r)\pi$, $-rE' - (1-r)I$
Dishonest $(1-p)$	$r\pi^* + (1-r)\pi'$, $-E$	0, $-rE' - (1-r)I'$

Expert: *Honest* $\succ$ *Dishonest*

$\Rightarrow$



Application: expert diagnosis

Cost of major repair $>$ expected cost of self-repair

Mixed strategy equilibrium when $E > rE' + (1-r)I'$

Exp		Consumer: *Accept* (q)	Consumer: *Reject* $(1-q)$
	Honest (p)	$r\pi^* + (1-r)\pi$, $-rE - (1-r)I$	$(1-r)\pi$, $-rE' - (1-r)I$
	Dishonest $(1-p)$	$r\pi^* + (1-r)\pi'$, $-E$	0, $-rE' - (1-r)I'$

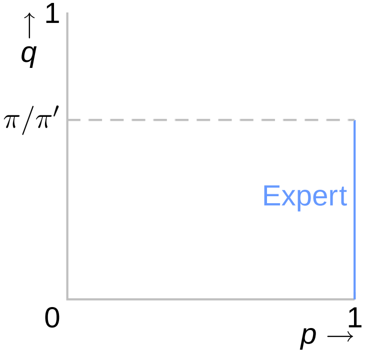
Expert: *Honest* $\succ$ *Dishonest*

$\Rightarrow$

$$q(r\pi^* + (1-r)\pi) + (1-q)(1-r)\pi > q(r\pi^* + (1-r)\pi')$$

$\Rightarrow$

$$q < \pi/\pi'$$



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Consumer

Exp	*Accept* (q)	*Reject* $(1-q)$
Honest (p)	$r\pi^* + (1-r)\pi,\ -rE-(1-r)I$	$(1-r)\pi,\ -rE'-(1-r)I$
Dishonest $(1-p)$	$r\pi^* + (1-r)\pi',\ -E$	$0,\ -rE'-(1-r)I'$

Expert: *Honest* $\sim$ *Dishonest*

$\Rightarrow$

$$q(r\pi^*+(1-r)\pi)+(1-q)(1-r)\pi = q(r\pi^*+(1-r)\pi')$$

$\Rightarrow$

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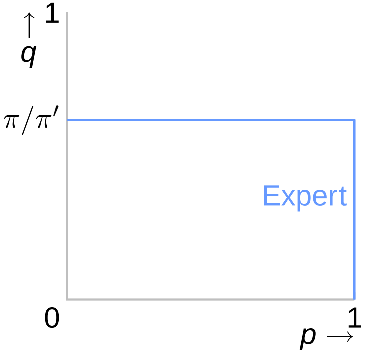

Application: expert diagnosis

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Mixed strategy equilibrium when $E > rE' + (1-r)I'$

Consumer

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Honest (p)	$r\pi^* + (1-r)\pi, -rE - (1-r)I$	$(1-r)\pi, -rE' - (1-r)I$
Dishonest $(1-p)$	$r\pi^* + (1-r)\pi', -E$	$0, -rE' - (1-r)I'$

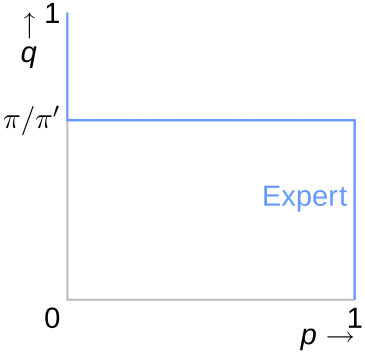
Expert: *Honest* $\prec$ *Dishonest*

$\Rightarrow$

$$q(r\pi^*+(1-r)\pi)+(1-q)(1-r)\pi < q(r\pi^* + (1-r)\pi')$$

$\Rightarrow$

$$q > \pi/\pi'$$



Application: expert diagnosis

Cost of major repair $>$ expected cost of self-repair

Mixed strategy equilibrium when $E > rE' + (1-r)I'$

		Consumer	
		Accept (q)	*Reject* $(1-q)$
Exp	*Honest* (p)	$r\pi^* + (1-r)\pi\,,\ -rE - (1-r)I$	$(1-r)\pi\,,\ -rE' - (1-r)I$
	Dishonest $(1-p)$	$r\pi^* + (1-r)\pi'\,,\ -E$	$0\,,\ -rE' - (1-r)I'$

Consumer: *Accept* $\succ$ *Reject*

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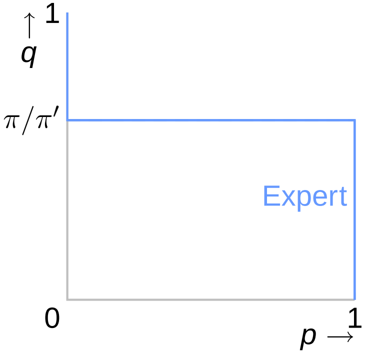

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Cost of major repair $>$ expected cost of self-repair

Mixed strategy equilibrium when $E > rE' + (1-r)I'$

Consumer

		Accept (q)	Reject $(1-q)$
Exp	Honest (p)	$r\pi^* + (1-r)\pi\,,\ -rE - (1-r)I$	$(1-r)\pi\,,\ -rE' - (1-r)I$
	Dishonest $(1-p)$	$r\pi^* + (1-r)\pi'\,,\ -E$	$0\,,\ -rE' - (1-r)I'$

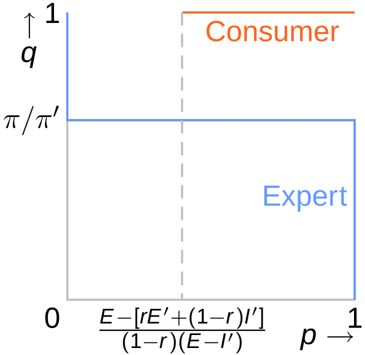
Consumer: *Accept* $\succ$ *Reject*

$\Rightarrow$

$$p(-rE-(1-r)I)+(1-p)(-E) > p(-rE'-(1-r)I) + (1-p)(-rE'-(1-r)I')$$

$\Rightarrow$

$$p > \frac{E - [rE' + (1-r)I']}{(1-r)(E-I')}$$



Application: expert diagnosis

Cost of major repair $>$ expected cost of self-repair

Mixed strategy equilibrium when $E > rE' + (1-r)I'$

Consumer

Exp	Accept (q)	Reject ($1-q$)
Honest (p)	$r\pi^* + (1-r)\pi\,,\ -rE-(1-r)I$	$(1-r)\pi\,,\ -rE'-(1-r)I$
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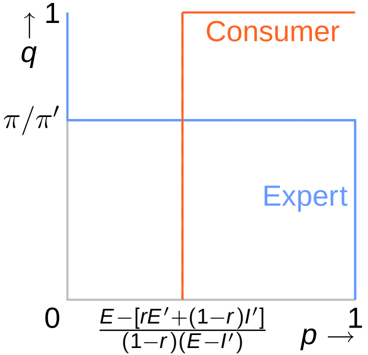
Consumer: *Accept* $\sim$ *Reject*

$\Rightarrow$

$$p(-rE-(1-r)I)+(1-p)(-E) = p(-rE'-(1-r)I) + (1-p)(-rE'-(1-r)I')$$

$\Rightarrow$

$$p = \frac{E-[rE'+(1-r)I']}{(1-r)(E-I')}$$



Application: expert diagnosis

Cost of major repair $>$ expected cost of self-repair

Mixed strategy equilibrium when $E > rE' + (1-r)I'$

		Consumer	
		Accept (q)	*Reject* $(1-q)$
Exp	*Honest* (p)	$r\pi^* + (1-r)\pi\,,\ -rE-(1-r)I$	$(1-r)\pi\,,\ -rE'-(1-r)I$
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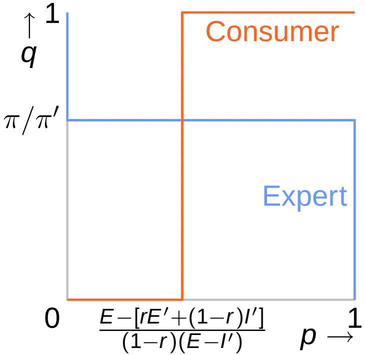
Consumer: *Accept* $\prec$ *Reject*

$\Rightarrow$

$$p(-rE-(1-r)I)+(1-p)(-E) < p(-rE'-(1-r)I) + (1-p)(-rE'-(1-r)I')$$

$\Rightarrow$

$$p < \frac{E-[rE'+(1-r)I']}{(1-r)(E-I')}$$



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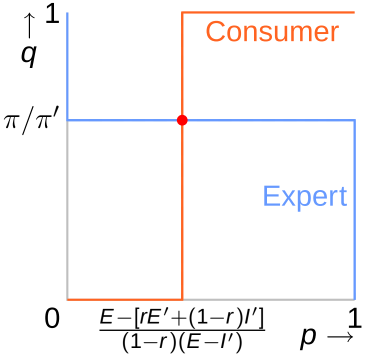
Unique Nash equilibrium with

$$p = \frac{E - [rE' + (1-r)I']}{(1-r)(E-I')}$$

$$q = \pi/\pi'$$



Application: expert diagnosis

Mixed strategy equilibrium when $E > rE' + (1-r)I'$

$$p = \frac{E - [rE' + (1-r)I']}{(1-r)(E-I')}$$

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Application: expert diagnosis

Mixed strategy equilibrium when $E > rE' + (1-r)I'$

$$p = \frac{E - [rE' + (1-r)I']}{(1-r)(E-I')}$$
$$q = \pi/\pi'$$

We have $p > 0$ and $0 < q < 1$.

Application: expert diagnosis

Mixed strategy equilibrium when $E > rE' + (1-r)I'$

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$$p = \frac{E - [rE' + (1-r)I']}{(1-r)(E-I')} = 1 - \frac{r(E'-E)}{(1-r)(E-I')}$$

Application: expert diagnosis

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so $p < 1$.

Application: expert diagnosis

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Hence equilibrium in which

- some experts are honest, some dishonest

Application: expert diagnosis

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so $p < 1$.

Hence equilibrium in which

- some experts are honest, some dishonest
- some consumers accept major diagnoses (“credulous”), some reject them (“wary”)

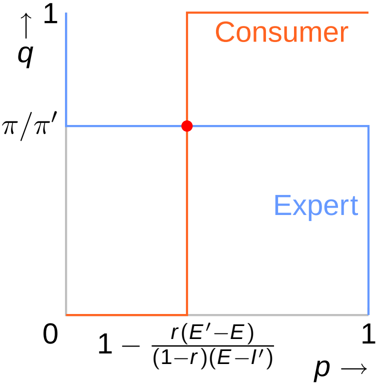
Application: expert diagnosis

Mixed strategy equilibrium: comparative statics

$$\text{prob. expert honest} = p = 1 - \frac{r(E' - E)}{(1-r)(E - I')}$$

$$\text{prob. consumer accepts major diagnosis} = q = \pi/\pi'$$

- Major problems less common (more reliable cars) $\Rightarrow r \downarrow$



Application: expert diagnosis

Mixed strategy equilibrium: comparative statics

$$\text{prob. expert honest} = p = 1 - \frac{r(E' - E)}{(1-r)(E - I')}$$

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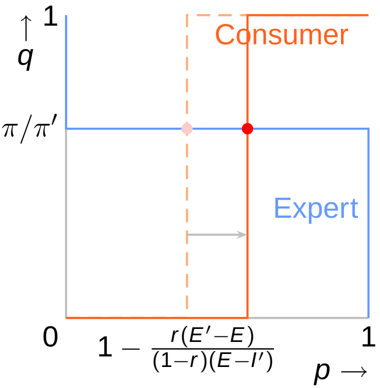
- Major problems less common (more reliable cars) $\Rightarrow r \downarrow$
  - $\Rightarrow$ $p \uparrow$, q unchanged
  - $\Rightarrow$ more experts honest, consumer behavior unchanged



Application: expert diagnosis

Mixed strategy equilibrium: comparative statics

$$\text{prob. expert honest} = p = 1 - \frac{r(E' - E)}{(1 - r)(E - I')}$$

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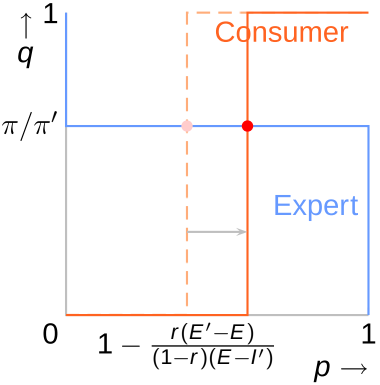
- Major problems less common (more reliable cars) $\Rightarrow r \downarrow$
  - $\Rightarrow$ $p \uparrow$, q unchanged
  - $\Rightarrow$ more experts honest, consumer behavior unchanged
  - intuition: major problems less common $\Rightarrow$ consumer has less to lose from ignoring expert's advice, so probability of expert being honest must rise for her advice to be heeded



Application: expert diagnosis

Mixed strategy equilibrium: comparative statics

$$\text{prob. expert honest} = p = 1 - \frac{r(E' - E)}{(1 - r)(E - I')}$$

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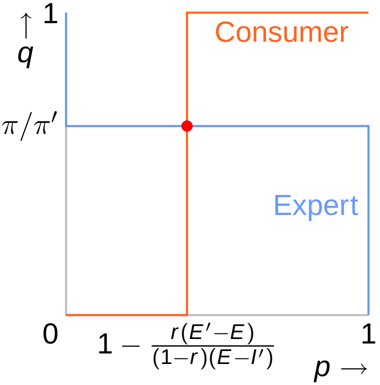
- Major repairs less expensive relative to minor ones (technical advance?) $\Rightarrow E \downarrow$



Application: expert diagnosis

Mixed strategy equilibrium: comparative statics

$$\text{prob. expert honest} = p = 1 - \frac{r(E' - E)}{(1-r)(E - I')}$$

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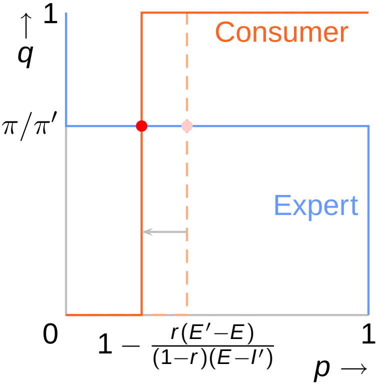
- Major repairs less expensive relative to minor ones (technical advance?) $\Rightarrow E \downarrow$
  - $\Rightarrow$ $p \downarrow$, q unchanged
  - $\Rightarrow$ fewer experts honest, consumer behavior unchanged



Application: expert diagnosis

Mixed strategy equilibrium: comparative statics

$$\text{prob. expert honest} = p = 1 - \frac{r(E' - E)}{(1-r)(E - I')}$$

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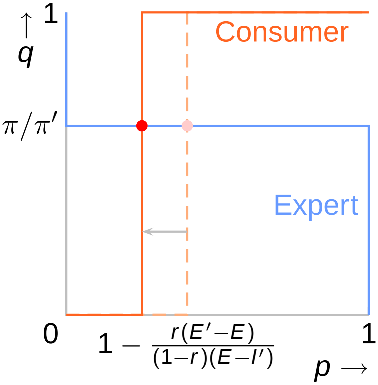
- Major repairs less expensive relative to minor ones (technical advance?) $\Rightarrow E \downarrow$
  - $\Rightarrow$ $p \downarrow$, q unchanged
  - $\Rightarrow$ fewer experts honest, consumer behavior unchanged
  - intuition: major repairs less costly $\Rightarrow$ consumer has more to lose from ignoring expert's advice, so she heeds the advice even if experts are less likely to be honest



Application: expert diagnosis

Mixed strategy equilibrium: comparative statics

$$\text{prob. expert honest} = p = 1 - \frac{r(E' - E)}{(1-r)(E - I')}$$

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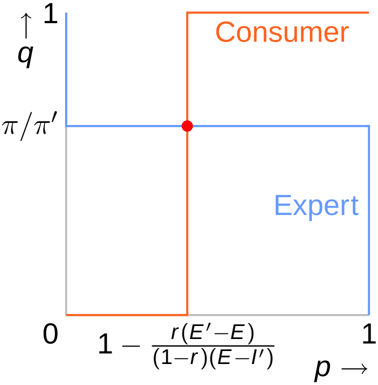
- $\pi' \downarrow$ (better regulation, so that fraud is harder): q increases



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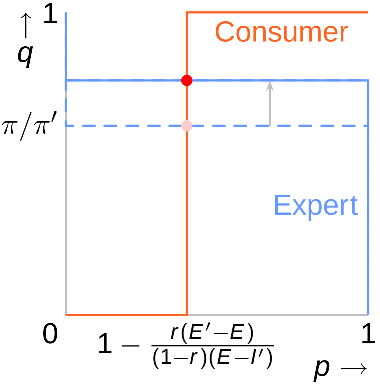
  - $\Rightarrow$ $q \uparrow$, p unchanged
  - $\Rightarrow$ consumers are less wary—they are more likely to accept diagnoses



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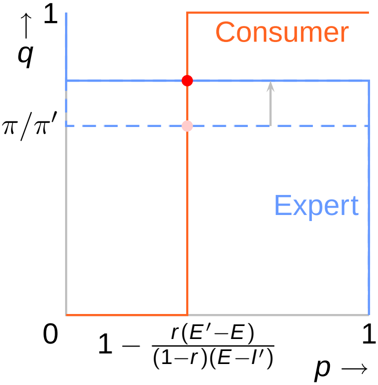
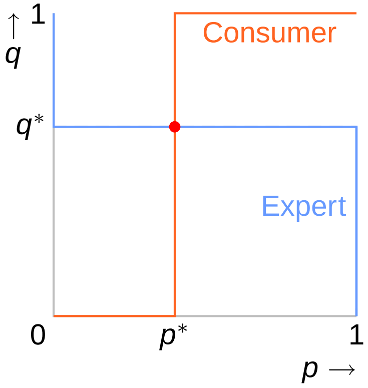
- $\pi' \downarrow$ (better regulation, so that fraud is harder): q increases
  - $\Rightarrow$ $q \uparrow$, p unchanged
  - $\Rightarrow$ consumers are less wary—they are more likely to accept diagnoses
  - intuition: experts have less to gain from being dishonest, so it pays for them to be dishonest only if consumers are less wary (note: fraud unchanged!)



Application: expert diagnosis

Mixed strategy equilibrium: possible dynamics

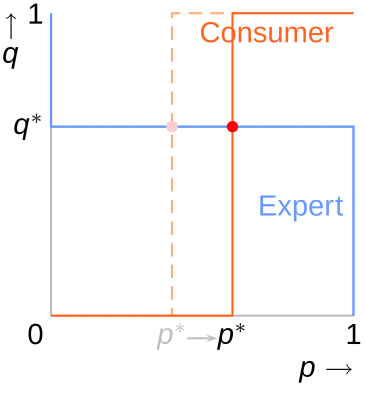
- Start at equilibrium



Application: expert diagnosis

Mixed strategy equilibrium: possible dynamics

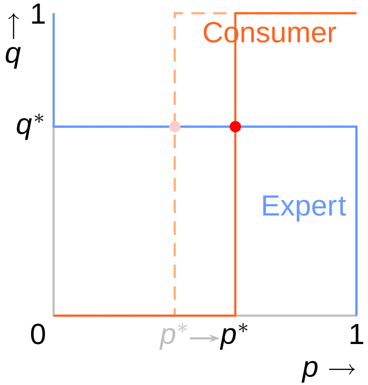
- Start at equilibrium
- Parameter changes



Application: expert diagnosis

Mixed strategy equilibrium: possible dynamics

- Start at equilibrium
- Parameter changes $\Rightarrow$ how is new equilibrium reached?

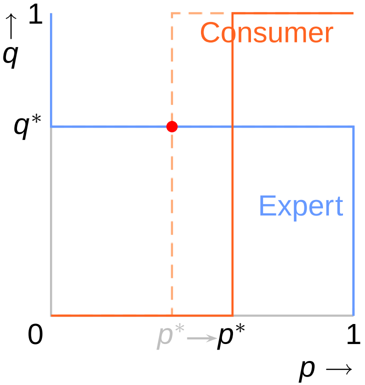


Application: expert diagnosis

Mixed strategy equilibrium: possible dynamics

- Start at equilibrium
- Parameter changes $\Rightarrow$ how is new equilibrium reached?

- $r \downarrow \Rightarrow$ consumer's best response function shifts right

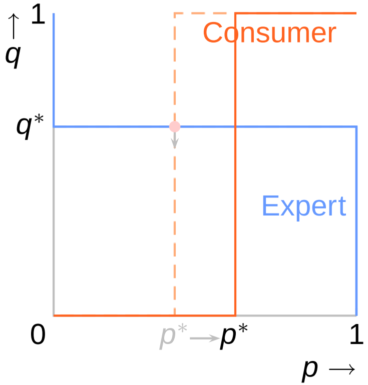


Application: expert diagnosis

Mixed strategy equilibrium: possible dynamics

- Start at equilibrium
- Parameter changes $\Rightarrow$ how is new equilibrium reached?

- $r \downarrow \Rightarrow$ consumer's best response function shifts right
- Given old p^*, best q is now 0, so q starts decreasing

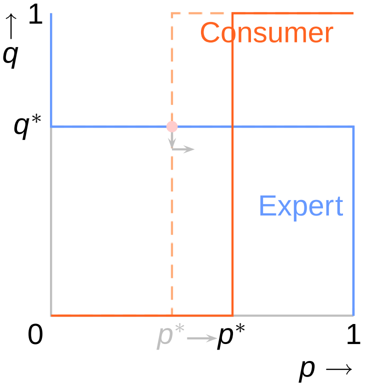


Application: expert diagnosis

Mixed strategy equilibrium: possible dynamics

- Start at equilibrium
- Parameter changes $\Rightarrow$ how is new equilibrium reached?

- $r \downarrow \Rightarrow$ consumer's best response function shifts right
- Given old p^*, best q is now 0, so q starts decreasing
- When q decreases, best p is 1, so p starts increasing



Application: expert diagnosis

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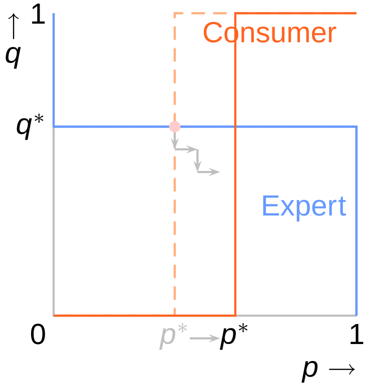
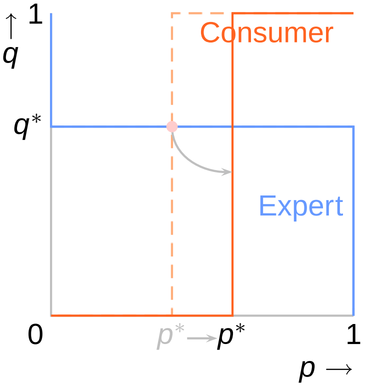


Application: expert diagnosis

Mixed strategy equilibrium: possible dynamics

- Start at equilibrium
- Parameter changes $\Rightarrow$ how is new equilibrium reached?

- $r \downarrow \Rightarrow$ consumer's best response function shifts right
- Given old p^*, best q is now 0, so q starts decreasing
- When q decreases, best p is 1, so p starts increasing
- As long as $p <$ new p^*, best q is 0, so q decreases



Application: expert diagnosis

Mixed strategy equilibrium: possible dynamics

- Start at equilibrium
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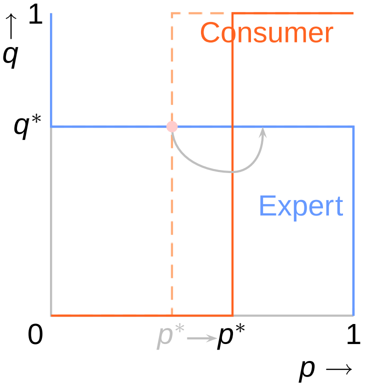
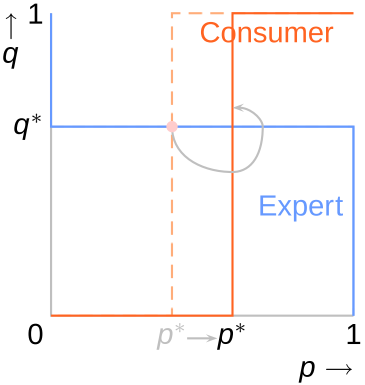


Application: expert diagnosis

Mixed strategy equilibrium: possible dynamics

- Start at equilibrium
- Parameter changes $\Rightarrow$ how is new equilibrium reached?

- Once $p >$ new p^*, best q is 1, so q increases
- When q increases above q^*, best p is zero, so p decreases

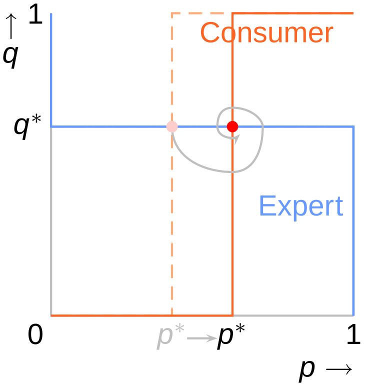


Application: expert diagnosis

Mixed strategy equilibrium: possible dynamics

- Start at equilibrium
- Parameter changes $\Rightarrow$ how is new equilibrium reached?

- Once $p >$ new p^*, best q is 1, so q increases
- When q increases above q^*, best p is zero, so p decreases
- Depending on adjustment speeds, new equilibrium may eventually be reached



Application: expert diagnosis

Nash equilibrium: summary

- Price of major repair less than expected cost of consumer fixing problem themselves $\Rightarrow$

Application: expert diagnosis

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Application: expert diagnosis

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Application: expert diagnosis

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Application: expert diagnosis

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  - major problems less common $\Rightarrow$ more experts honest, consumer behavior unaffected

Application: expert diagnosis

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- Comparative statics:
  - major problems less common $\Rightarrow$ more experts honest, consumer behavior unaffected
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Application: expert diagnosis

Nash equilibrium: summary

- Price of major repair less than expected cost of consumer fixing problem themselves $\Rightarrow$ all experts dishonest, all consumers credulous (pure strategy Nash equilibrium)
- Price of major repair greater than expected cost of consumer fixing problem themselves $\Rightarrow$ some experts dishonest, some honest; some consumers credulous, some wary (mixed strategy Nash equilibrium)
- Comparative statics:
  - major problems less common $\Rightarrow$ more experts honest, consumer behavior unaffected
  - major repairs less expensive $\Rightarrow$ fewer experts honest, consumer behavior unaffected
  - less profit from major repair of minor problem $\Rightarrow$ consumers less wary, expert behavior unaffected

Application: reporting a crime (“volunteer’s dilemma”)

- Many people witness a crime

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- One person's reporting crime to police suffices
- When deciding whether to report, each person doesn't know whether anyone else has reported
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- If the crime is reported, everyone obtains benefit $v > c$
- How many people report? How does number depend on size of group?

Application: reporting a crime ("volunteer's dilemma")

Strategic game

Players n individuals

Application: reporting a crime (“volunteer’s dilemma”)

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Actions For each player, $\{Call, Don't\ call\}$

Application: reporting a crime (“volunteer’s dilemma”)

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Payoffs For each player i,

$$u_i(a) = \begin{cases} & \text{if } a_i = \mathit{Call} \\ & \text{if } a_i = \mathit{Don't\ call} \text{ and} \\ & \quad a_j = \mathit{Call} \text{ for some } j \neq i \\ & \text{if } a_j = \mathit{Don't\ call} \text{ for all } j \end{cases}$$

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Application: reporting a crime (“volunteer’s dilemma”)

Nash equilibria

- Equilibria in pure strategies?

Application: reporting a crime (“volunteer’s dilemma”)

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  - How can these equilibria be realized? For an equilibrium in which player 1 calls, who is player 1?
- Look for *symmetric* equilibrium in *mixed* strategies

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Application: reporting a crime (“volunteer’s dilemma”)

Mixed strategy Nash equilibrium

Conclusion: game has a symmetric mixed strategy Nash equilibrium, in which every player calls with probability

$$p = 1 - (c/v)^{1/(n-1)}$$

(Note: this number is between 0 and 1.)

Application: reporting a crime ("volunteer's dilemma")

Mixed strategy Nash equilibrium: comparative statics

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- Probability that at least one person calls:

$$\begin{aligned}\Pr\{\text{at least one person calls}\} \\ &= 1 - \Pr\{\text{no one calls}\}\end{aligned}$$

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Because $n \uparrow \Rightarrow p \downarrow$,

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$\Rightarrow$ the more people, the *less* likely the police are informed!

Application: reporting a crime ("volunteer's dilemma")

Summary

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  - More generally, in a large group a collectively beneficial action is less likely to be taken than in a small one
  - For example, result suggests that a broken streetlight is less likely to be reported if it is outside an apartment block than if it is in an area of low-density housing

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$\Updownarrow$

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