Economics 316

Fall 2017 Martin J. Osborne

Solutions to Problem Set 4

1. The players' best response functions are shown in Figure 1.

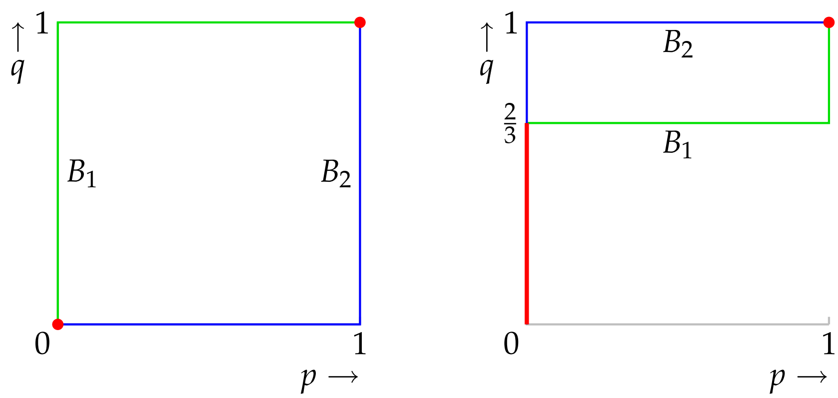

Figure 1. The players' best response functions in the left game (left panel) and right game (right panel) in Problem 1. The probability that player 1 assigns to T is p and the probability that player 2 assigns to L is q. The disks and the heavy line indicate Nash equilibria.

   Left game: two mixed strategy Nash equilibria (both of which are in fact pure), $((0,1),(0,1))$ and $((1,0),(1,0))$.

   Right game: $((1,0),(1,0))$ and any strategy pair $((0,1),(q,1-q))$ for $0 \le q \le \frac{2}{3}$.

2. Notice that if player 1 chooses X with probability 1 then player 2 is indifferent between L and R. Thus every mixed strategy of player 2 is a best response to X. If, for the mixed strategy player 2 chooses, player 1's expected payoffs to T and B are no more than her expected payoff to X, then we have an equilibrium.

   Denote the probability with which player 2 chooses L by q. Then player 1's expected payoff to T is q and her expected payoff to B is $2(1-q)$. Thus the conditions for her expected payoff to T and B to be at most her expected payoff to X are $q \le 1$ and $2(1-q) \le 1$, or $q \ge \frac{1}{2}$.

   We conclude that every pair of mixed strategies $((0,0,1),(q,1-q))$ for which $\frac{1}{2} \le q \le 1$ is a mixed strategy Nash equilibrium of the game. (You were asked only for two equilibria; any two that satisfy these conditions are adequate to answer the question.)

3. (a) The players' best response functions are shown in Figure 3. The

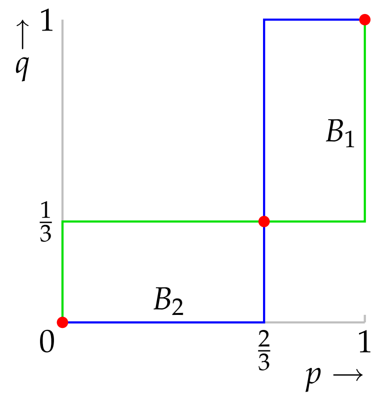

Figure 2. The players' best response functions in the game Problem 3. The probability that player 1 assigns to T is p and the probability that player 2 assigns to L is q. The disks indicates the Nash equilibria.

game has three Nash equilibria: $((1,0),(1,0))$, $((\frac{2}{3},\frac{1}{3}),(\frac{1}{3},\frac{2}{3}))$, and $((0,1),(0,1))$.

(b) Given the equilibrium strategy $(\frac{1}{3},\frac{2}{3})$ of player 2 in the game in part (a), player 1's payoff to the action X is less than her payoff to the actions B and S, so a Nash equilibrium of this game is $((\frac{2}{3},\frac{1}{3},0),(\frac{1}{3},\frac{2}{3}))$.

4. Top game: indicated strategy pair is a mixed strategy Nash equilibrium (expected payoffs to M and B for player 1 each equal to $\frac{7}{4}$ and expected payoff to T less ($\frac{3}{4}$); expected payoffs to L and C for player 2 each equal to 3 and expected payoff to R also 3).

   Bottom game: indicated strategy pair is not a mixed strategy Nash equilibrium because player 2's expected payoff to C exceeds her expected payoffs to L and R.

5. (a)

	Rock	*Paper*	*Scissors*
Rock	$0, \ \ 0$	$-1, \ \ 1$	$1, -1$
Paper	$1, -1$	$0, \ \ 0$	$-1, \ \ 1$
Scissors	$-1, \ \ 1$	$1, -1$	$0, \ \ 0$

(b) Given the symmetry of the game, it is plausible that the strategy pair in which each player assigns probability $\frac{1}{3}$ to each action is an equilibrium. To show that indeed this strategy pair is an equilibrium, calculate each player's expected payoff to each of her actions if the other player assigns probability $\frac{1}{3}$ to each of her actions. These expected payoffs are the same, equal to zero. Thus the strategy pair is a mixed strategy Nash equilibrium.

(c) Player 2's action *Rock* is strictly dominated. The remaining game has a unique mixed strategy equilibrium, in which player 1 chooses *Rock* with probability $\frac{1}{3}$ and *Paper* with probability $\frac{2}{3}$, and player 2 chooses *Paper* with probability $\frac{2}{3}$ and *Scissors* with probability $\frac{1}{3}$. The equilibrium payoff of player 1 is $-\frac{1}{3}$ and that of player 2 is $\frac{1}{3}$.