Economics 316

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Solutions for Problem Set 3

1. The unique Nash equilibrium remains (m, m); the argument is exactly the same as before. (The dividing line between the supporters of two candidates with different positions changes. If $x_i < x_j$, for example, the dividing line is $\frac{1}{3}x_i + \frac{2}{3}x_j$ rather than $\frac{1}{2}(x_i + x_j)$. This change does not affect the argument for Nash equilibrium.)

2. (a) Both parties choose $(0,0)$; both parties choose $(1,0)$: Nash equilibria.

   Both parties choose $(0,1)$; both parties choose $(1,1)$: not Nash equilibria. (In each case, they tie; either of them can deviate to $(0,0)$ and win.)

   One party chooses $(0,0)$, one party chooses $(1,0)$: Nash equilibrium.

   One party chooses $(0,0)$, one party chooses $(1,1)$: Not Nash equilibrium. (Party at $(1,1)$ loses, and can deviate to $(1,0)$ and tie.)

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   One party chooses $(1,0)$, one party chooses $(1,1)$: Not Nash equilibrium. (Party at $(1,1)$ loses, and can deviate to $(1,0)$ (or $(0,0)$) and tie.)

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   One party chooses $(1,1)$, one party chooses $(0,1)$: Not Nash equilibrium. (Parties tie; party at $(0,1)$ can deviate to $(0,0)$ and win.)

   (b) (i) If one party chooses $(0,0)$ and the other chooses $(1,0)$ then the parties tie (each receives half of the votes). If the party at $(1,0)$ deviates to $(0.25, 0.25)$ then it wins because it obtains the votes of all citizens whose favorite positions are $(0,1)$, $(1,1)$, or $(1,0)$. Thus the pair of actions is not a Nash equilibrium.

(ii) If both parties choose $(\frac{1}{2}, 0)$ then they tie. If one party deviates to $(0.25, 0.25)$ then it obtains the votes of all citizens whose favorite positions are $(0, 0)$, $(0, 1)$, or $(1, 1)$, so that it wins. Thus the pair of actions is not a Nash equilibrium.

(In fact, the game has no Nash equilibrium.)

3. The pair of actions is not a Nash equilibrium because player 1 can move slightly to the *left*, so that she still wins, and increase her payoff because her policy is now closer to her favorite position than is m.

4. Suppose that the favorite position of the single candidate is $m - d$. Then her payoff is $b - c$. If she exits, her payoff is K, which is less than $b - c$ by assumption.

   Consider a citizen whose favorite position x satisfies $|m - x| > d$. If this citizen enters, she loses and does not affect the outcome, so it is optimal for her to stay out.

   Finally, consider a citizen whose favorite x satisfies $|m - x| < d$. If this citizen enters, she wins and obtains the payoff $b - c$. The citizen with the most to gain is the one whose favorite position is close to $m + d$. Her payoff if she does not enter is close to $-2d$ (the position of the winner is $m - d$) and her payoff if she enters is $b - c$ (because in this case she wins). Thus her payoff from entering is higher than her payoff from staying out if $b - c > -2d$. Thus if $d \leq \frac{1}{2}(c - b)$, no citizen can gain by entering.

   We conclude that the game has a Nash equilibrium in which a single citizen enters if the entrant's favorite position is within the distance $\frac{1}{2}(c - b)$ of m.

5. No, the game has no such equilibrium. For two citizens to stand as candidates, we need $b > 2c$, in which case a third citizen with a favorite position close to m who stands wins and obtains a positive payoff.

6. The outcome is that candidates 1 and 3 tie for first place and candidate 2 loses (the votes of the citizens with favorite position 0.6 go to candidate 2), so that the policy is 0 with probability $\frac{1}{2}$ and 1 with probability $\frac{1}{2}$.

   To check whether the action profile is a Nash equilibrium, consider each candidate in turn.

Candidate 1 Her payoff currently is $\frac{1}{2}b + \frac{1}{2}(-1) - c$. If she exits, candidate 2 wins (with 60% of the votes) and her payoff becomes $-\frac{1}{4}$.

Candidate 2 Her payoff currently is $\frac{1}{2}(-\frac{1}{4}) + \frac{1}{2}(-\frac{3}{4}) - c$. If she exits, candidate 3 wins (with 55% of the votes) and her payoff becomes $-\frac{3}{4}$.

Candidate 3 Her payoff currently is $\frac{1}{2}b + \frac{1}{2}(-1) - c$. If she exits, candidate 2 wins (with 60% of the votes) and her payoff becomes $-\frac{3}{4}$.

If candidate 1 does not want to exit, then candidate 3 also does not want to exit, so for no candidate to want to exit we need

$$\frac{1}{2}b - c \geq \frac{1}{4}$$
$$-\frac{1}{2} - c \geq -\frac{3}{4}$$

or $b \geq \frac{1}{2} + 2c$ and $c \leq \frac{1}{4}$.

Now consider the entry of any citizen as a candidate.

- If the citizen's favorite position is 0, she splits the votes of the citizens with favorite position 0 with candidate 1, causing candidate 3 to win. This outcome is worse for her than candidates 1 and 3 each winning with probability $\frac{1}{2}$.
- If the citizen's favorite position between 0 and 0.25, she gets no votes and does not affect the outcome; given that she pays the cost of entry, she is worse off.
- If the citizen's favorite position is 0.25, she splits the votes of the citizens with favorite position 0.25 with candidate 2 and thus has no effect on the outcome; as in the previous case she is worse off.
- If the citizen's favorite position is between 0.25 and 1, she gets 15% of the votes and does not affect the outcome; once again she is worse off.
- If the citizen's favorite position is 1, she splits the votes of the citizens with favorite position 1 with candidate 3, causing candidate 1 to win. This outcome is worse for her than candidates 1 and 3 each winning with probability $\frac{1}{2}$.

We conclude that if $b \geq \frac{1}{2} + 2c$ and $c \leq \frac{1}{4}$ then the action profile is a Nash equilibrium.