Economics 316

Fall 2017 Martin J. Osborne

Problem Set 3

1. Consider a variant of Hotelling's model with two parties in which the voters' preferences are asymmetric. Specifically, suppose that each voter cares twice as much about policy differences to the left of her favorite position than about policy differences to the right of her favorite position. For example, suppose that the position of party 1 is 0.2 and the position of party 2 is 0.5. Then a citizen whose favorite position is 0.35 prefers party 2 to party 1, and a citizen whose favorite position is 0.3 is exactly indifferent between the two parties (because the distance between her favorite position and the position of party 1 is exactly half of the distance between her favorite position and the position of party 2).

   How does the Nash equilibrium differ from the Nash equilibrium of Hotelling's model?

2. Consider a variant of Hotelling's model of electoral competition in which a policy has two dimensions rather than one. (You can think of one dimension as "economic policy" and the other as "social policy", for example.) In each dimension, a policy can taken any value from 0 to 1. Thus the policy space consists of the square with corners $(0,0)$, $(1,0)$, $(0,1)$, and $(1,1)$. Assume that the favorite policy of every citizen is one of the corners of the square. Specifically, $(0,0)$ is the favorite policy of 30% of citizens, $(1,0)$ is the favorite policy of 30% of citizens, $(0,1)$ is the favorite policy of 20% of citizens, and $(1,1)$ is the favorite policy of 20% of citizens.

   Suppose that there are two parties. Each citizen votes for the party whose policy is closest to the citizen's favorite policy. The votes of citizens whose favorite positions are equidistant from the policies of the two parties are split equally between the parties. For example, if party 1's position is $(0,0)$ and party 2's is $(1,0)$, party 1 gets the votes of all citizens whose favorite positions are $(0,0)$ or $(0,1)$ and party 2 gets the votes of all citizens whose favorite positions are $(1,0)$ or $(1,1)$.

(a) Find the Nash equilibria of the strategic game that models this situation whenthe only policies either party can choose are $(0,0)$, $(1,0)$, $(0,1)$, and $(1,1)$.

(b) If each party can choose any point in the square (that is, any (x, y) with $0 \leq x \leq 1$ and $0 \leq y \leq 1$), determine whether (i) the action pair in which one party chooses $(0,0)$ and the other chooses $(1,0)$ is a Nash equilibrium and (ii) the action pair in which both parties choose $(\frac{1}{2}, 0)$ is a Nash equilibrium.

3. In class I discussed the variant of Hotelling's model in which the two parties care *only* about the position of the winner (and not at all about winning). I considered the case in which party 1's favorite position is to the left of the median, m, of the citizens' favorite positions and party 2's favorite position is to the right of m. I claimed that in this case, the only Nash equilibrium is the action pair in which both parties choose m. Why is the action pair in which party 1 chooses m and party 2 chooses her favorite position not a Nash equilibrium?

4. Consider the citizen-candidate model. Assume $b < c$. Does the game have a Nash equilibrium in which exactly one candidate enters and does so at a position different from m?

5. Does the citizen-candidate model have a Nash equilibrium in which there are two candidates, both of whose favorite positions are m?

6. An interesting possibility in the citizen-candidate model is that there is a Nash equilibrium in which a candidate loses. To make the argument simple, consider a very special distribution of preferences that is very different from the ones considered in class. The range of possible positions is from 0 to 1. Forty percent of citizens have favorite position 0, 5% have favorite position 0.25, 15% have favorite position 0.6, and 40% have favorite position 1. (See Figure 1. No citizen has a favorite position different from 0, 0.25, 0.6, and 1.) Suppose that three citizens enter as candidates: one ("candidate 1") with favorite position 0, one ("candidate 2") with favorite position 0.25, and one ("candidate 3") with favorite position 1. What is the outcome? Are there values of b and c for which the action profile is a Nash equilibrium?

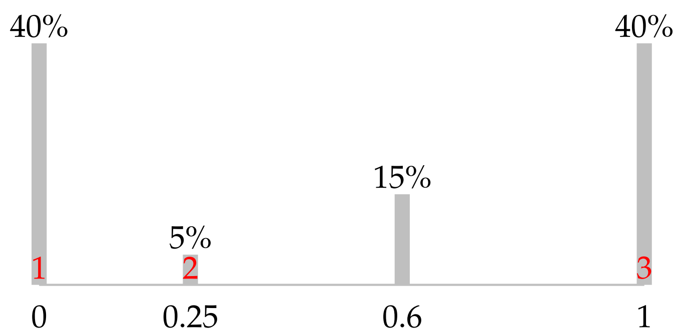

Figure 1. The distribution of the citizens' favorite positions in Problem 6. The positions of the three candidates indicated in red.