Economics 316

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Solutions to problems for Tutorial 3

1. (a) If the products are different, then either firm increases its market share by making its product more similar to that of its rival. Thus in every possible equilibrium the products are the same.

   If $x_1 = x_2 \neq m$ then each firm's market share is 50%. If either firm changes its product to be closer to m then its market share rises above 50%.

   Thus the only possible equilibrium is $(x_1, x_2) = (m, m)$. This pair of positions is an equilibrium, since each firm's market share is 50%, and if either firm changes its product its market share falls below 50%.

   (b) If all firms' products are the same, each obtains one-third of the market.

   If $x_1 = x_2 = x_3 = m$ then any firm, by changing its product a little, can obtain close to one-half of the market.

   If $x_1 = x_2 = x_3 \neq m$ then any firm, by changing its product a little, can obtain more than one-half of the market.

   If the firms' products are not all the same, then at least one of the extreme products is different from the other two products, and the firm that produces it can increase its market share by making it more similar to the other products.

   Thus when there are three firms there is no Nash equilibrium.

2. The game has four Nash equilibria: $(0,0)$, $(-\frac{1}{2}, \frac{1}{2})$, $(-\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$.

   If $(x_1, x_2) = (0, 0)$ then the parties tie and the outcome is 0 and each party's payoff is $-\frac{1}{2}$. If either party deviates, it loses, and the outcome remains the same. Thus neither party can profitably deviate.

   If $(x_1, x_2) = (-\frac{1}{2}, \frac{1}{2})$ then the parties tie and the outcome is (by assumption) 0. Party 1's payoff is $-\frac{1}{2}$. If it deviates to 0 then it wins outright, so the outcome is 0, and its payoff remains $-\frac{1}{2}$. If it deviates to -1 or 1 then it loses and the outcome does not change. If it deviates to $\frac{1}{2}$ then it ties, the outcome is $\frac{1}{2}$, and its payoff is -1. Thus it has no profitable deviation. The same argument applies to party 2.

If $(x_1, x_2) = (-\frac{1}{2}, 0)$ then party 2 wins, so that the outcome is 0 and consequently party 1's payoff is $-\frac{1}{2}$ and party 2's payoff is also $-\frac{1}{2}$. If party 1 deviates to 0 it ties and the outcome remains 0, so its payoff is unchanged. If it deviates to -1, $\frac{1}{2}$, or 1, it loses, so that its payoff also remains the same. If party 2 deviates to $\frac{1}{2}$ then it ties, the outcome remains 0, and its payoff remains $-\frac{1}{2}$. If it deviates to $-\frac{1}{2}$ then it ties, the outcome is $-\frac{1}{2}$, and its payoff is -1. If it deviates to -1 or 1, it loses, so that the outcome and its payoff remain the same. Thus neither player has a profitable deviation.

The argument regarding $(x_1, x_2) = (0, \frac{1}{2})$ is symmetric with the argument for $(x_1, x_2) = (-\frac{1}{2}, 0)$.

No other pair of positions is a Nash equilibrium for the following reasons.

- If $x_1 = -1$ and $x_2 = 1$ then the parties tie and party 1 can increase its payoff by deviating to $-\frac{1}{2}$.
- If $x_1 = -1$ and $x_2 = \frac{1}{2}$ then party 2 wins and party 1 can increase its payoff by deviating to $-\frac{1}{2}$.
- If $x_1 = -1$ and $x_2 = 0$ or $x_2 = -\frac{1}{2}$ then party 2 wins and the outcome is 0. Party 2 can increase its payoff by deviating to $\frac{1}{2}$.
- Similar arguments apply to the cases in which $x_2 = 1$.

3. Yes, the game has such an equilibrium. Suppose that one candidate (say candidate 2) enters at m (the median), one (candidate 1) at $m - k$, and one (candidate 3) at $m + k$, where the fraction of citizens with favorite positions less than $m - \frac{1}{2}k$ is $\frac{1}{3}$ and the fraction of citizens with favorite positions greater than $m + \frac{1}{2}k$ is also $\frac{1}{3}$.

   The outcome is that the three candidates tie. The policy implemented is thus $m - k$ with probability $\frac{1}{3}$, m with probability $\frac{1}{3}$, and $m + k$ with probability $\frac{1}{3}$. Given $b = c = 0$, the payoff of candidate 1 is $\frac{1}{3} \cdot 0 + \frac{1}{3} \cdot (-k) + \frac{1}{3} \cdot (-2k) = -k$ (because the distance between the policies of candidates 1 and 2 is k and the distance between the policies of candidates 1 and 3 is $2k$). The payoff of candidate 2 is $\frac{1}{3} \cdot (-k) + \frac{1}{3} \cdot 0 + \frac{1}{3} \cdot (-k) = -\frac{2}{3}k$.

If candidate 1 exits, candidate 2 becomes the winner, so that the policy implemented is m. Hence candidate 1's payoff is $-k$ (the distance between her favorite position and m). That is, her payoff is unchanged if she exits. A similar argument applies to candidate 3.

If candidate 2 exits, candidates 1 and 3 each win with probability $\frac{1}{2}$, do that candidate 2's payoff is $\frac{1}{2} \cdot (-k) + \frac{1}{2} \cdot (-k) = -k$. Thus she is worse off.

If a citizen with favorite position between $m - k$ and m enters, candidate 3 becomes the winner, making the entering citizen worse off. A similar argument applies to the entry of a citizen whose favorite position is between m and $m + k$.

If a citizen with favorite position of at most $m - k$ enters, candidates 2 and 3 tie, making the entering citizen worse off. A similar argument applies to the entry of a citizen whose favorite position is a least $m + k$.

If a citizen with favorite position equal to m enters, candidates 1 and 3 tie, and the entering citizen is worse off.

4. The action profile in which all three candidates choose m, the median of the citizens' favorite positions, is a Nash equilibrium. (In fact, it is the only equilibrium.)

   For this action profile, each candidate wins with probability $\frac{1}{3}$.

   If a candidate deviates to a position different from m, she wins on the first round, and faces each of the other candidates with probability $\frac{1}{2}$ on the second round. In each of the second round contests, she loses.

   If a candidate withdraws, her payoff becomes zero.