Economics 316

Fall 2017 Martin J. Osborne

Problems for Tutorial 3

1. Consider a variant of Hotelling's model of electoral competition that models the choices of product characteristics by competing firms. The set of positions is the range of possible characteristics for a product, and the citizens are consumers rather than voters. Consumers' tastes differ; each consumer buys (at a fixed price, possibly zero) one unit of the product she likes best. Each firm's objective is to maximize its market share. (Note that this objective differs from the objective of the parties in the model of electoral competition. A party wants to obtain more votes than the other parties, not to maximize its number of votes.)

   (a) Show that when there are two firms the unique Nash equilibrium is (m, m) (both firms offer the consumers' median favorite product).

   (b) Show that when there are three firms there is no Nash equilibrium. (Start by arguing that when there are two firms whose products differ, either firm is better off making its product more similar to that of its rival.)

2. Consider a variant of the model of party positioning in which each party cares about the winning position. Instead of assuming that any number can be chosen as a position, assume that the only positions possible are -1, $-\frac{1}{2}$, 0, $\frac{1}{2}$, and 1. Assume also that there are two parties; party 1's favorite position is $-\frac{1}{2}$ and party 2's favorite position is $\frac{1}{2}$. Assume that party 1's payoff for the policy x is $-|x - (-\frac{1}{2})|$ and party 2's payoff for the policy x is $-|x - \frac{1}{2}|$. Assume finally that the distribution of the citizens' favorite positions is symmetric about 0, so that if, for example, one candidate's position is -1 and the other's is 1, or one candidate's position is $-\frac{1}{2}$ and the other's is $\frac{1}{2}$, the candidates tie.

   Find the Nash equilibria of the strategic game that models this situation.

3. Consider the citizen-candidate model in the case that $b = c = 0$ and the distribution of the citizens' favorite positions is single-peaked and symmetric (as in the last part of the class). Does the game have a Nash equilibrium in which three candidates enter at different positions and all win with positive probability?

4. Consider a variant of Hotelling's model of electoral competition with *three* candidates that differs from the one studied in class only in that the winner is determined by a "runoff" rule rather than plurality rule. Assume (as in class) that the distribution of the citizens' favorite positions has a unique median and (as in class for the case of three candidates) that one option for each candidate is to not compete, which the candidate regards as better than entering and losing but worse than entering and winning with probability $\frac{1}{3}$.

   Precisely, there are potentially two rounds of voting. If, in the first round, one candidate obtains more than half of the votes, then she wins (and there is no second round). Otherwise, a second vote is held between the *two* candidates who obtained the most votes in the first round, and the candidate who obtains the most votes on the second round wins the election.

   If several candidates are tied for first place on the first round, each pair of them goes to the second round with the same probability. For example, if candidates 1, 2, and 3 are tied for first place on the first round, then the second round is between 1 and 2 with probability $\frac{1}{3}$, between 1 and 3 with probability $\frac{1}{3}$, and between 2 and 3 with probability $\frac{1}{3}$. If one candidate, say candidate 1, wins on the first round, but does not get more than half of the votes, and there is a tie for second place between candidates 2 and 3, then the second round is between 1 and 2 with probability $\frac{1}{2}$ and between 1 and 3 with probability $\frac{1}{2}$. If the two candidates in the *second* round are tied, each of them wins with probability $\frac{1}{2}$.

   If two or more candidates share the same position, they share equally the votes generated by that position.

   Find a Nash equilibrium of the strategic game that models this situation.