# ECO316: Applied game theory

## Lecture 3

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# Electoral competition

Parties choose platforms

↓

Each citizen decides whether to vote and if so for which party

↓

Government determined by electoral system

How well does elected government reflect citizens' preferences?

How does electoral system affect elected government?

# Electoral competition

Initially assume 2 parties, exogenous

Number of parties?

How to model platform?

Platform is number

Parties choose platforms

Objectives of parties?

To win (don't care about platform)

↓

Criteria for decision?

Each citizen decides whether to vote and if so for which party

Vote for party with platform like best (non-strategic)

↓

Government determined by electoral system

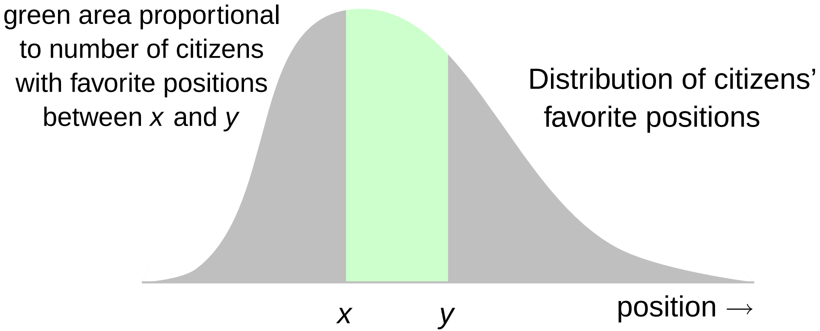
Nature of electoral system?

Majority rule in single district

# Electoral competition: Hotelling's model

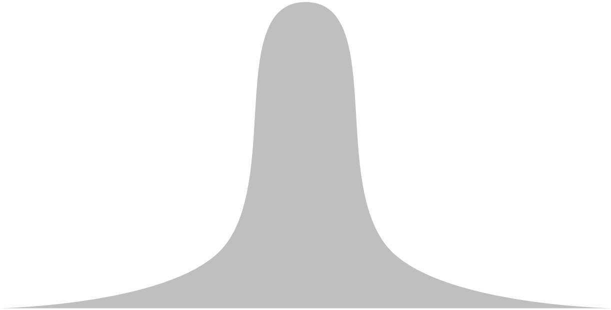
## Model

- Political position is number
- Set of positions represents left–right spectrum
- Each citizen has favorite position
- Large number of citizens



# Electoral competition: Hotelling's model

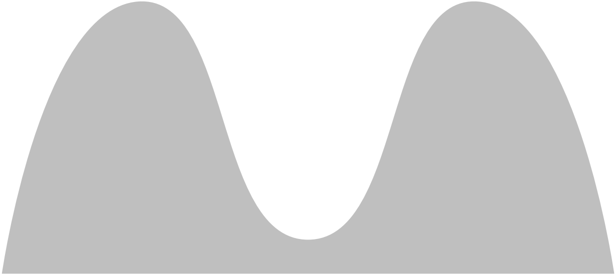
## Examples of distributions of citizens' preferences



Few extremists, most citizens favor centrist position

# Electoral competition: Hotelling's model

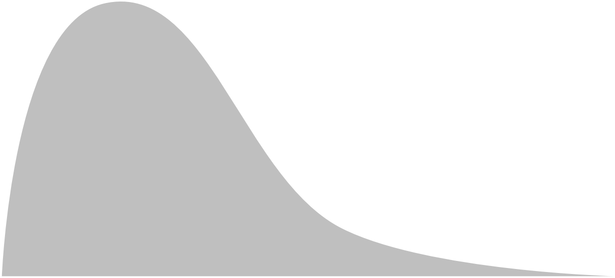
## Examples of distributions of citizens' preferences



Many extremists on both sides, few centrists

# Electoral competition: Hotelling's model

## Examples of distributions of citizens' preferences

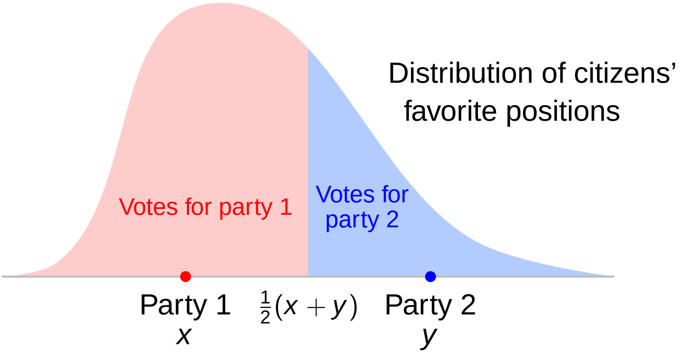


Many citizens with favorite positions on left,
few with favorite positions on right

# Electoral competition: Hotelling's model

## Model

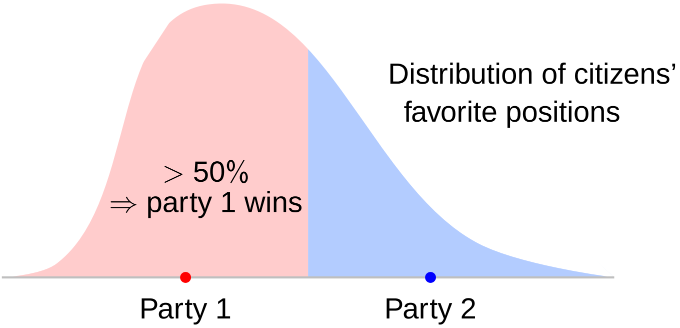
- Each party chooses position
- Each citizen votes for party with position closest to her favorite position—that is, she votes *sincerely*



# Electoral competition: Hotelling's model

## Model

- Party who obtains most votes wins
- Each party cares only about winning; *no party has ideological attachment to any position*



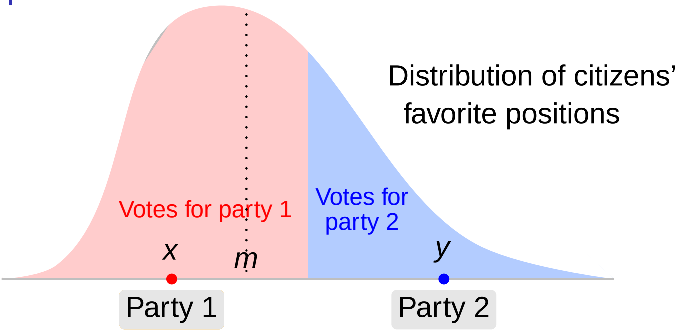
# Electoral competition: Hotelling's model

## Strategic game

- Players: parties
- For each party,
  - possible actions: positions
  - preferences: win $\succ$ tie $\succ$ lose

# Hotelling's model with two parties

## Nash equilibrium

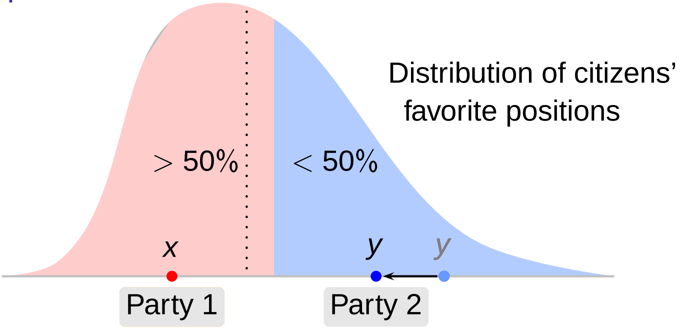



Equilibrium with parties at $x$ and $y$?

- Party 2 loses

# Hotelling's model with two parties

## Nash equilibrium

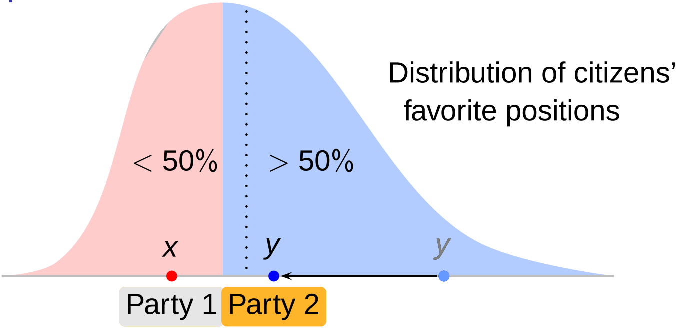



Equilibrium with parties at $x$ and $y$?

- Party 2 loses
- If party 2 moves left, its vote share increases

# Hotelling's model with two parties

## Nash equilibrium



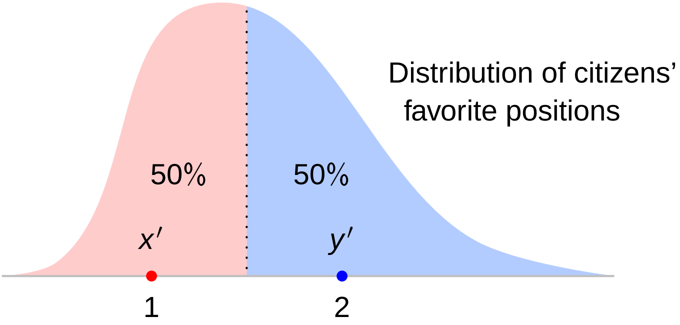

Equilibrium with parties at $x$ and $y$?

- Party 2 loses
- If party 2 moves left, its vote share increases
- If party 2 moves far enough left, it wins

$\Rightarrow$ not Nash equilibrium

# Hotelling's model with two parties

## Nash equilibrium



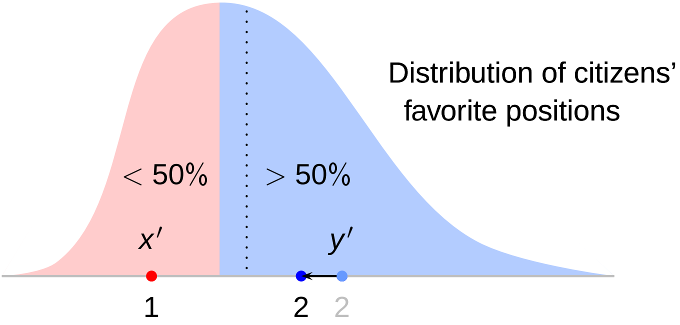

Equilibrium with parties at $x'$ and $y'$?

- Parties tie
- Party 2 can move slightly left and win

$\Rightarrow$ not Nash equilibrium

# Hotelling's model with two parties

## Nash equilibrium




Equilibrium with parties at $x'$ and $y'$?

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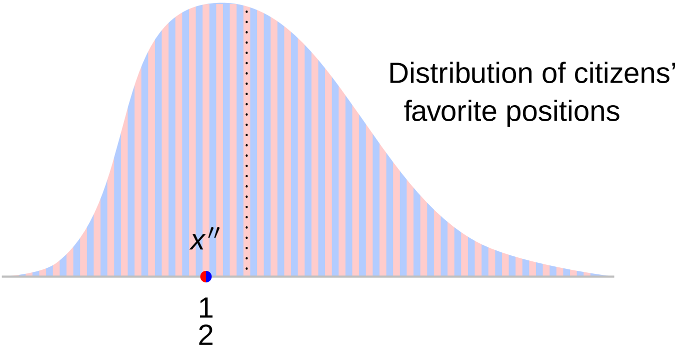
$\Rightarrow$ not Nash equilibrium

# Hotelling's model with two parties

## Nash equilibrium



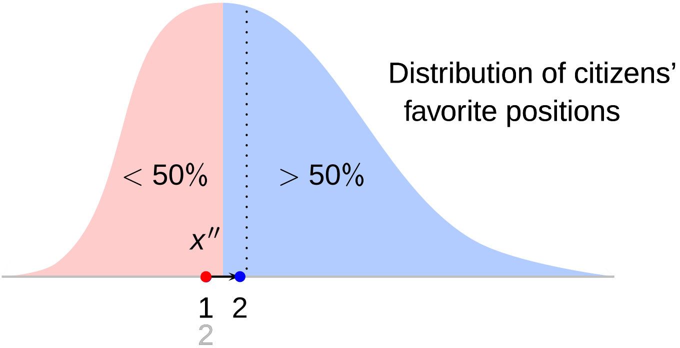

Equilibrium with both parties at $x''$?

- Parties tie
- Party 2 (for example) can deviate slightly to right and win

⇒ not Nash equilibrium

# Hotelling's model with two parties

## Nash equilibrium



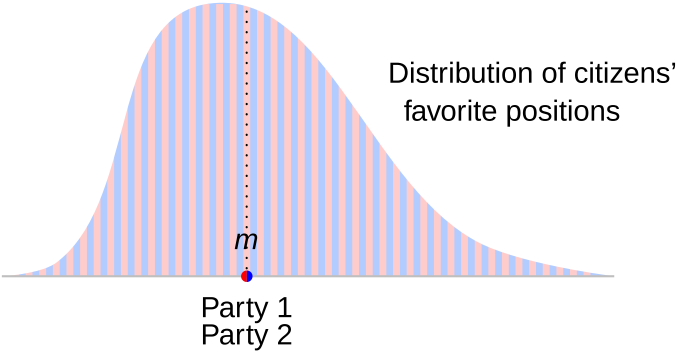

Equilibrium with both parties at $x''$?

- Parties tie
- Party 2 (for example) can deviate slightly to right and win

$\Rightarrow$ not Nash equilibrium

# Hotelling's model with two parties

## Nash equilibrium



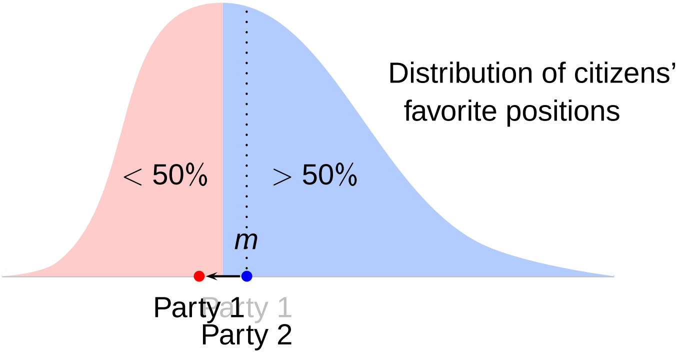

Equilibrium with both parties at $m$?

- Parties tie
- If either party deviates, it loses

$\Rightarrow$ Nash equilibrium

# Hotelling's model with two parties

## Nash equilibrium




Equilibrium with both parties at $m$?

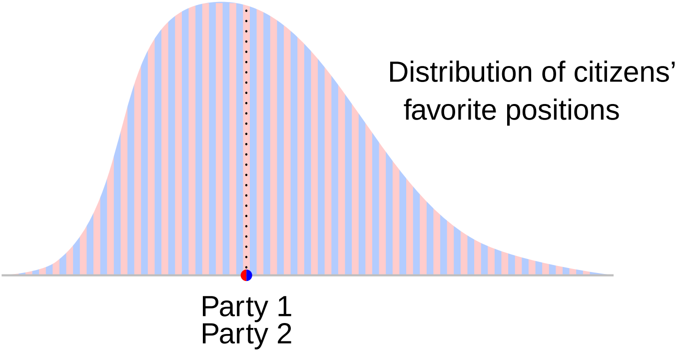
- Parties tie
- If either party deviates, it loses

$\Rightarrow$ Nash equilibrium

# Hotelling's model with two parties

## Nash equilibrium: conclusion

Unique Nash equilibrium, in which both parties choose *median* favorite position of citizens



# Hotelling's model with two parties

## Nash equilibrium with two parties: Proof

- *Median* favorite position: position $m$ such that exactly half of citizens' favorite positions are $\leq m$ and half are $\geq m$. (Assume distribution of favorite positions is such that there is only one position with this property.)
- $(m, m)$ is an equilibrium: results in tie; if either party chooses position different from $m$, then it loses
- No other pair of positions is a Nash equilibrium:
  - If one party loses, it can do better by moving to $m$, where it wins outright if opponent's position $\neq m$ and ties for first place if opponent's position $= m$
  - If parties tie (because their positions are either the same or symmetric about $m$), either party can do better by moving to $m$, where it wins outright

# Hotelling's model with two parties

- Parties don't generally adopt same position
- What ingredient is missing from model?
- Parties should care about position, not only about winning?
- Consider case in which each party cares *only* about the *position of the winning party*
- Assume that if parties tie for votes, policy is average of parties' positions

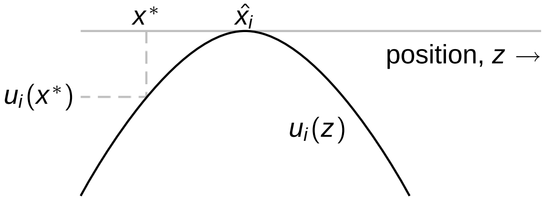
# Parties that care about winning position

## Strategic game

- Players: two parties
- For each party $i$,
  - possible actions: positions
  - payoff: $u_i(x^*)$ … inner (or average of winners' positions if tied) and $u_i$ has single peak, at $\hat{x}_i$

Favorite position of party $i$

$i$'s payoff when policy of winner is $x^*$

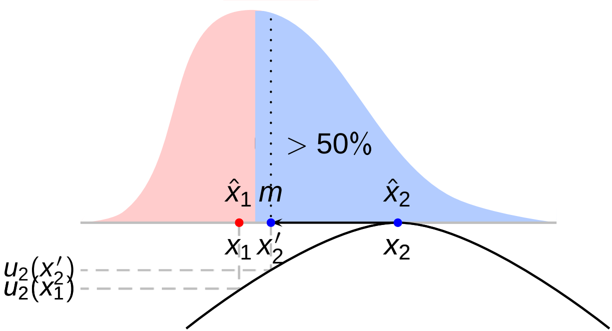



Payoff of party $i$, with favorite position $\hat{x}_i$

Assume $\hat{x}_1 < m < \hat{x}_2$ (one party on left and one on right)

# Parties that care about winning position

## Nash equilibrium




Equilibrium in which each party chooses its favorite position?

- Suppose positions such that party 1 wins

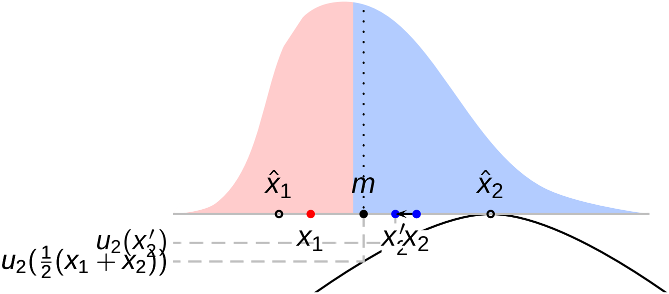
$\Rightarrow$ party 2's payoff $u_2(x_1)$

- Party 2 moves to $m \Rightarrow$ wins and gets $u_2(x_2') > u_2(x_1)$

$\Rightarrow$ not Nash equilibrium

# Parties that care about winning position

## Nash equilibrium



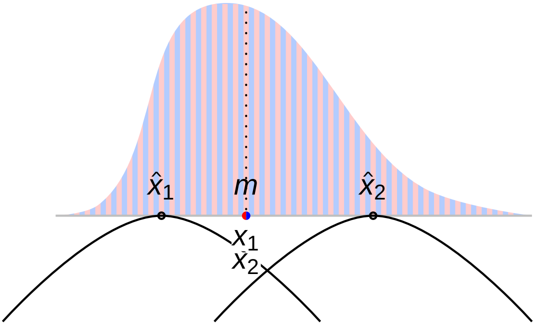

Equilibrium in which parties tie and moderate their positions?

- Outcome is $\frac{1}{2}(x_1 + x_2) = m \Rightarrow$ party 2's payoff $u_2(\frac{1}{2}(x_1 + x_2)) = u_2(m)$
- Party 2 moves to left $\Rightarrow$ party 2 wins and gets payoff $u_2(x_2') > u_2(\frac{1}{2}(x_1 + x_2)) = u_2(m)$

$\Rightarrow$ not Nash equilibrium

# Parties that care about winning position

## Nash equilibrium




Equilibrium in which parties both choose median position?

- Outcome is $m \Rightarrow$ party $i$'s payoff $u_i(m)$
- Either party moves $\Rightarrow$ loses $\Rightarrow$ outcome unchanged

$\Rightarrow$ Nash equilibrium!

# Parties that care about winning position

If we check all possible configurations of positions we find . . .

## Nash equilibrium

Parties care only about winning position $\Rightarrow$ game has unique Nash equilibrium, in which both parties choose median of citizens' favorite positions

That is: Nash equilibrium outcome is same as in case in which parties care only about winning!

# Parties that care about winning position

- Intuitively, when party moves closer to its rival it faces tradeoff:
  - higher probability of winning
  - less desirable position if wins
- Model doesn't capture tradeoff because probability of winning is either 1 or 0 (or parties tie)
- To capture tradeoff, need to add uncertainty (e.g. about citizens' preferences)
- If do so, model becomes difficult to work with, but can yield equilibrium with distinct positions for parties

# Hotelling's model: three parties

## Three parties

- Return to model in which parties care only about winning, and consider case of *three* parties
- Suppose each party has option of staying out (quitting), which is better than losing (and worse than tying)

## Strategic game

- Players: three parties
- For each party,
  - possible actions: $\{Out\} \cup$ set of possible positions
  - preferences: win $\succ$ tie $\succ$ *Out* $\succ$ lose

# Hotelling's model: three parties

## Three parties: Nash equilibrium

**Claim** In Nash equilibrium, a party that runs either wins outright or ties for first place—it does not lose

**Argument** Party that runs and loses can quit, which it prefers to losing

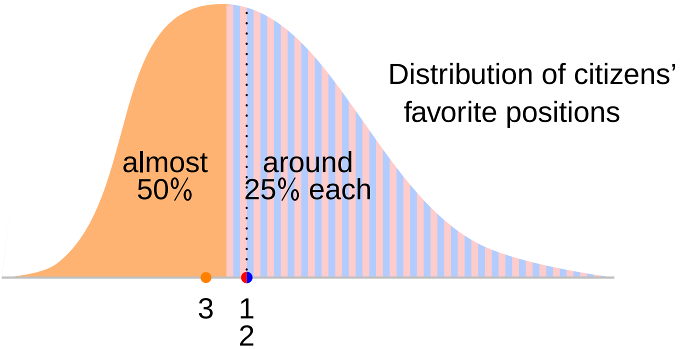
Now consider possible configurations:

- **No party enters** Not equilibrium: a party can deviate and enter, and win
- **One party enters** Not equilibrium: another party can enter at same position and tie for first place

# Hotelling's model: three parties

## Three parties: Nash equilibrium

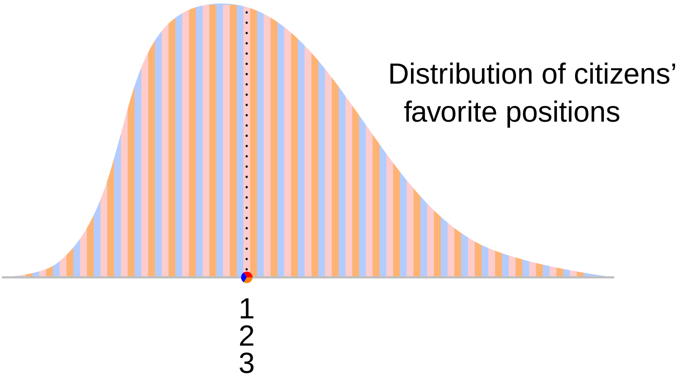
- **Two parties enter**
  - Must both choose median (by argument in two-party game)
  - But then third party can enter near median and win—so not Nash equilibrium



# Hotelling's model: three parties

## Three parties: Nash equilibrium

- **Three parties enter**
  - all choose median ⇒ they tie
  - one party deviates a little ⇒ it wins
  - so not Nash equilibrium



# Hotelling's model: three parties

## Three parties: Nash equilibrium

- **Three parties enter**
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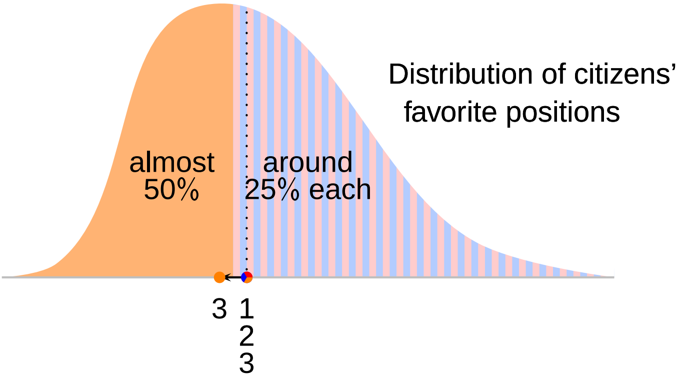
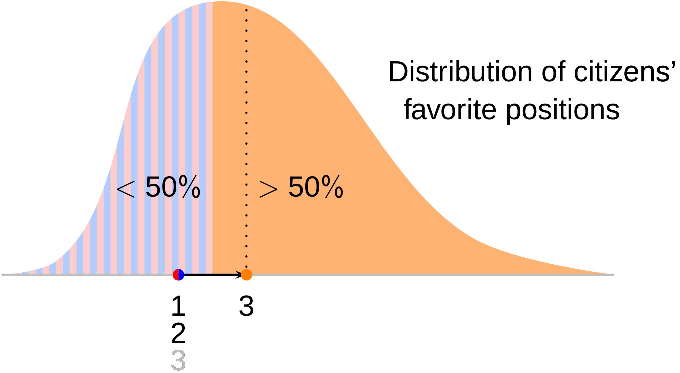


# Hotelling's model: three parties

## Three parties: Nash equilibrium

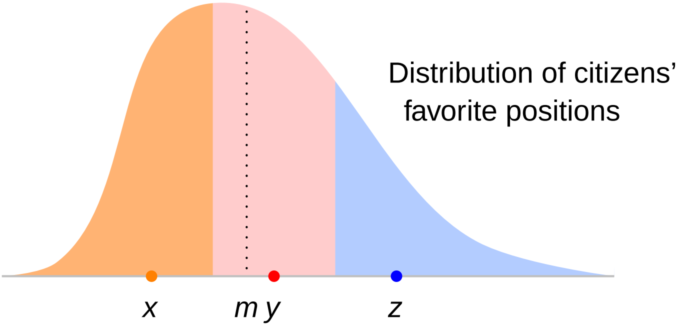
- **Three parties enter**
  - all choose same position, $\neq$ median $\Rightarrow$ they tie
  - one party deviates to median $\Rightarrow$ it wins
  - so not Nash equilibrium



# Hotelling's model: three parties

## Three parties: Nash equilibrium

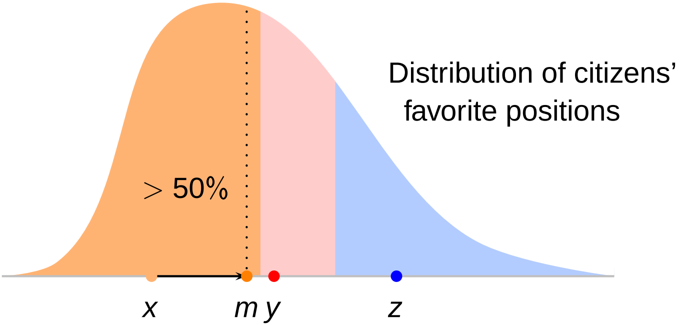
- **Three parties enter**
  - choose different positions $\Rightarrow$ must tie (else would exit)
  - Suppose positions $x < m < y < z$



# Hotelling's model: three parties

## Three parties: Nash equilibrium

- **Three parties enter**
  - choose different positions $\Rightarrow$ must tie (else would exit)
  - Suppose positions $x < m < y < z$ $\Rightarrow$ party at $x$ can move to $m$ and win outright
  - Not Nash equilibrium



# Hotelling's model: three parties

## Three parties: Nash equilibrium

- **Three parties enter**
  - choose different positions $\Rightarrow$ must tie (else would exit)
    - $x < y < m < z \Rightarrow$ party at $z$ can move to $m$ and win outright
    - $x < y = m < z \Rightarrow$ party at $x$ can move close to $m$ and win outright
    - $x = y < m < z \Rightarrow$ party at $z$ can move to $m$ and win outright
    - $x = y = m < z \Rightarrow$ party at $z$ can move close to $m$ and win outright
    - Note that $x < y < z \leq m$ is not possible, because party at $z$ then wins outright

## Conclusion

The game has no Nash equilibrium!

# Summary

- Two parties whose only objective is to win $\Rightarrow$ both choose median of citizens' favorite positions
- Two "ideological" parties, who care only about position of winner $\Rightarrow$ both choose median of citizens' favorite positions
- Three parties whose only objective is to win $\Rightarrow$ no Nash equilibrium!
- So no model so far consistent with two parties at different positions, or with three parties

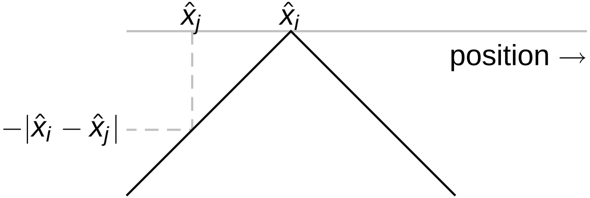
# Citizen-candidates

## Model

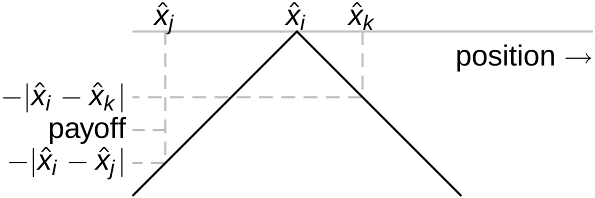
- Each citizen decides whether to become a candidate
- Decisions are made simultaneously
- *Candidates cannot hide their true preferences*; $i$'s favorite position is $\hat{x}_i$
- Running as a candidate entails a cost $c > 0$
- After entry decisions are made, citizens vote sincerely
- If candidates tie for first place, winner is selected randomly (with equal probabilities)
- Winner gets payoff $b > 0$ (in addition to payoff from winning position)

# Citizen-candidates

## Payoff of citizen *i*




citizen *i* does not enter; *j* wins




citizen *i* does not enter; *j* and *k* tie for most votes

# Citizen-candidates

## Strategic game

- Players: citizens
- For each citizen $i$,
  - possible actions: $\{Run, Out\}$
  - payoff:

$$\begin{cases} -|\hat{x}_i - \hat{x}_j| & \text{if } i \text{ chooses } Out \text{ and } j \text{ wins} \\ -|\hat{x}_i - \hat{x}_j| - c & \text{if } i \text{ chooses } Run \text{ and } j \text{ wins} \\ b - c & \text{if } i \text{ chooses } Run \text{ and } i \text{ wins} \\ \frac{1}{2}b - \frac{1}{2}|\hat{x}_i - \hat{x}_j| - c & \text{if } i \text{ chooses } Run \text{ and } i \text{ and } j \text{ tie for first place} \end{cases}$$

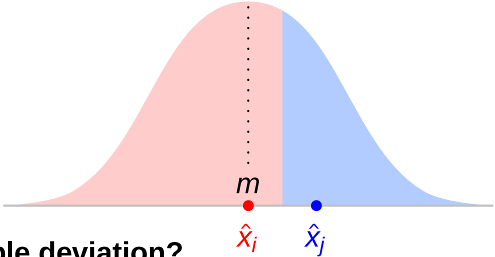
If no one enters, everyone's payoff is $K < b - c$.

*Assume symmetric single-peaked distribution of favorite positions* (makes some arguments easier)

# Citizen-candidates

## Nash equilibrium with one candidate?

Suppose citizen $i$ with favorite position $m$ is only candidate



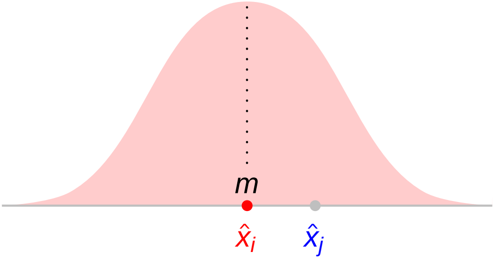

**Any profitable deviation?**

- $i$: current payoff $b - c$; withdraws $\Rightarrow$ payoff $K < b - c$
- another citizen with favorite position $m$: current payoff 0; enters $\Rightarrow$ payoff $\frac{1}{2}b - c \Rightarrow$ entry not profitable if $b \leq 2c$
- citizen $j$ with favorite position $\hat{x}_j \neq m$: current payoff $-|\hat{x}_j - m|$; enters $\Rightarrow$ loses $\Rightarrow$ payoff $-|\hat{x}_j - m| - c$

# Citizen-candidates

## Nash equilibrium with one candidate?

Suppose citizen $i$ with favorite position $m$ is only candidate




**Conclusion**

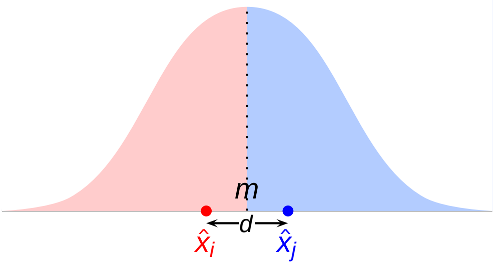
If $b \leq 2c$ the game has a Nash equilibrium with a single candidate whose position is $m$

Under some conditions the game also has an equilibrium with a single candidate whose position is different from $m$ (Exercise)

# Citizen-candidates

## Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie $\Rightarrow$ symmetric about $m$

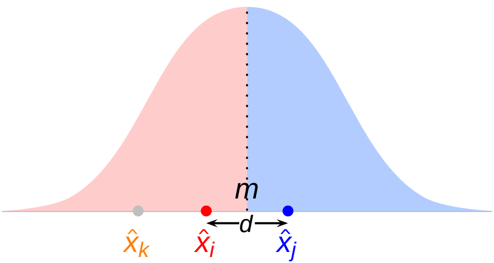



Let $d = \hat{x}_j - \hat{x}_i$ (distance between candidates' positions)

# Citizen-candidates

## Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie $\Rightarrow$ symmetric about $m$

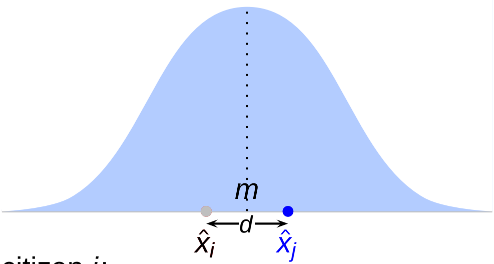



- Outcome is $\hat{x}_i$ with probability $\frac{1}{2}$ and $\hat{x}_j$ with probability $\frac{1}{2}$
- Payoff of $i$: $-\frac{1}{2}|\hat{x}_j - \hat{x}_i| + \frac{1}{2}b - c = -\frac{1}{2}d + \frac{1}{2}b - c$
- Payoff of $j$: $-\frac{1}{2}|\hat{x}_i - \hat{x}_j| + \frac{1}{2}b - c = -\frac{1}{2}d + \frac{1}{2}b - c$
- Payoff of any other citizen $k$: $-\frac{1}{2}|\hat{x}_k - \hat{x}_i| - \frac{1}{2}|\hat{x}_k - \hat{x}_j|$

# Citizen-candidates

## Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie ⇒ symmetric about $m$




Deviation by citizen $i$:

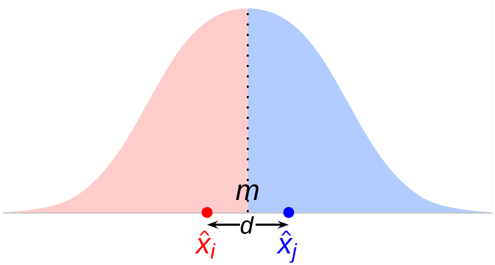
- Current payoff: $-\frac{1}{2}d + \frac{1}{2}b - c$
- Exit ⇒ outcome $\hat{x}_j$ ⇒ payoff $-|\hat{x}_j - \hat{x}_i| = -d$
- So for entry to be optimal,

$$-\frac{1}{2}d + \frac{1}{2}b - c \geq -d$$
$$\Rightarrow d \geq 2c - b$$

# Citizen-candidates

## Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie ⇒ symmetric about $m$

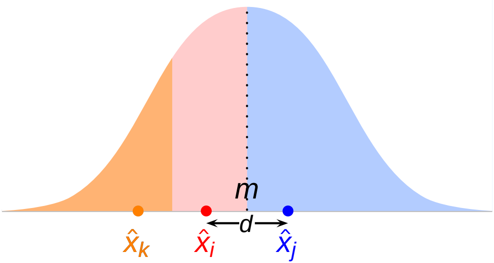



Deviation by citizen $j$: Same argument as for citizen $i$

# Citizen-candidates

## Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie ⇒ symmetric about $m$



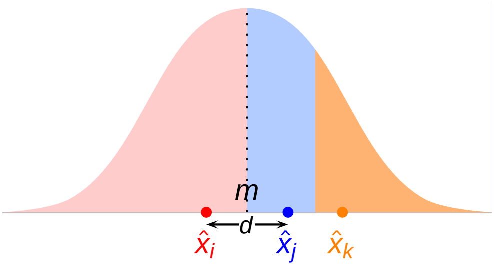

Deviation by citizen $k$ with favorite position $\hat{x}_k \leq \hat{x}_i$:

- Current payoff: $-\frac{1}{2}|\hat{x}_k - \hat{x}_i| - \frac{1}{2}|\hat{x}_k - \hat{x}_j|$
- Enter ⇒ winner is $j$
  ⇒ payoff $-|\hat{x}_j - \hat{x}_k| - c < -\frac{1}{2}|\hat{x}_k - \hat{x}_i| - \frac{1}{2}|\hat{x}_k - \hat{x}_j|$
  ⇒ $k$ is worse off

# Citizen-candidates

## Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie ⇒ symmetric about $m$

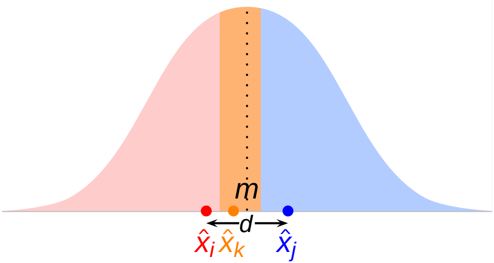



Entry by citizen $k$ with favorite position $\hat{x}_k \geq \hat{x}_j$: same argument

# Citizen-candidates

## Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie ⇒ symmetric about $m$



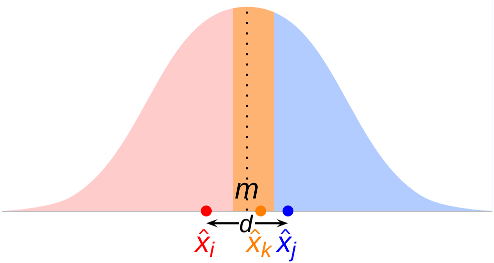

Entry by citizen $k$ with favorite position $\hat{x}_k$, where $\hat{x}_i < \hat{x}_k < m$:

- If $\hat{x}_i$ and $\hat{x}_j$ are close enough, $j$ wins
  ⇒ $k$ is worse off (because winning position is worse and pays entry cost $c$)

# Citizen-candidates

## Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie $\Rightarrow$ symmetric about $m$

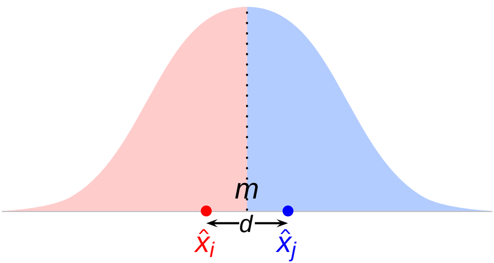



Deviation by citizen $k$ with favorite position $\hat{x}_k$, where $m < \hat{x}_k < \hat{x}_j$: same argument

# Citizen-candidates

## Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie ⇒ symmetric about $m$




**Conclusion** If distance between candidates is at least $2c - b$ but not big enough that a citizen who enters between them can win, the configuration is an equilibrium

# Citizen-candidates

## Nash equilibria with one and two candidates: summary

For a symmetric single-peaked distribution of favorite positions,

- if $b \leq 2c$ then there is an equilibrium with a single candidate
- there are equilibria with two candidates symmetrically located around the median favorite position, not too close together and not too far apart

# Electoral competition: summary

## Parties that care only about winning

**Two parties** Unique Nash equilibrium, in which position of both parties is median of citizens' favorite positions

**Three parties** No Nash equilibrium

## Parties that care only about winning policy

**Two parties** Unique Nash equilibrium, in which position of both parties is median of citizens' favorite positions

## Citizen-candidates

Nash equilibria with one, two, and more candidates. Equilibrium positions may be dispersed.