ECO316: Applied game theory

Lecture 3

Martin J. Osborne

Department of Economics
University of Toronto

2017.9.21

Table of contents

Electoral competition

Parties choose
platforms

Electoral competition

Parties choose
platforms

$\downarrow$

Each citizen decides whether to vote
and if so for which party

Electoral competition

Parties choose
platforms

↓

Each citizen decides whether to vote
and if so for which party

↓

Government determined by
electoral system

Electoral competition

Parties choose platforms

↓

Each citizen decides whether to vote and if so for which party

↓

Government determined by electoral system

How well does elected government reflect citizens' preferences?

Electoral competition

Parties choose platforms

↓

Each citizen decides whether to vote and if so for which party

↓

Government determined by electoral system

How well does elected government reflect citizens' preferences?

How does electoral system affect elected government?

Electoral competition

Number of parties?

Parties choose platforms

↓

Each citizen decides whether to vote and if so for which party

↓

Government determined by electoral system

Electoral competition

Initially assume 2 parties, exogenous

Number of parties?

Parties choose platforms

↓

Each citizen decides whether to vote and if so for which party

↓

Government determined by electoral system

Electoral competition

Initially assume 2 parties, exogenous

Number of parties?

How to model platform?

Parties choose platforms

↓

Each citizen decides whether to vote and if so for which party

↓

Government determined by electoral system

Electoral competition

Initially assume 2 parties, exogenous

Number of parties?

Platform is number

How to model platform?

Parties choose platforms

↓

Each citizen decides whether to vote and if so for which party

↓

Government determined by electoral system

Electoral competition

Initially assume 2 parties, exogenous

Number of parties?

Platform is number

How to model platform?

Parties choose platforms

Objectives of parties?

↓

Each citizen decides whether to vote and if so for which party

↓

Government determined by electoral system

Electoral competition

Initially assume 2 parties, exogenous

Number of parties?

Platform is number

How to model platform?

Parties choose platforms

Objectives of parties?

To win (don't care about platform)

↓

Each citizen decides whether to vote and if so for which party

↓

Government determined by electoral system

Electoral competition

Initially assume 2 parties, exogenous

Number of parties?

How to model platform?

Platform is number

Parties choose platforms

Objectives of parties?

To win (don't care about platform)

↓

Criteria for decision?

Each citizen decides whether to vote and if so for which party

↓

Government determined by electoral system

Electoral competition

Initially assume 2 parties, exogenous

Number of parties?

Platform is number

How to model platform?

Parties choose platforms

Objectives of parties?

To win (don't care about platform)

↓

Criteria for decision?

Each citizen decides whether to vote and if so for which party

Vote for party with platform like best (non-strategic)

↓

Government determined by electoral system

Electoral competition

Initially assume 2 parties, exogenous

Number of parties?

Platform is number

How to model platform?

Parties choose platforms

Objectives of parties?

To win (don't care about platform)

↓

Criteria for decision?

Each citizen decides whether to vote and if so for which party

Vote for party with platform like best (non-strategic)

↓

Government determined by electoral system

Nature of electoral system?

Electoral competition

Initially assume 2 parties, exogenous

Number of parties?

Platform is number

How to model platform?

Parties choose platforms

Objectives of parties?

To win (don't care about platform)

↓

Criteria for decision?

Each citizen decides whether to vote and if so for which party

Vote for party with platform like best (non-strategic)

↓

Government determined by electoral system

Nature of electoral system?

Majority rule in single district

Electoral competition: Hotelling's model

Harold Hotelling
1895–1973

Model

- Political position is number

position $\longrightarrow$

Electoral competition: Hotelling's model

Model

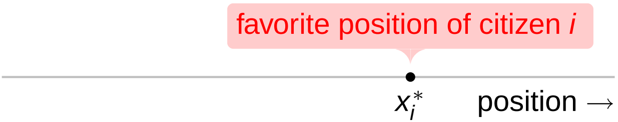
- Political position is number
- Set of positions represents left–right spectrum

position $\longrightarrow$

Electoral competition: Hotelling's model

Model

- Political position is number
- Set of positions represents left–right spectrum
- Each citizen has favorite position



Electoral competition: Hotelling's model

Model

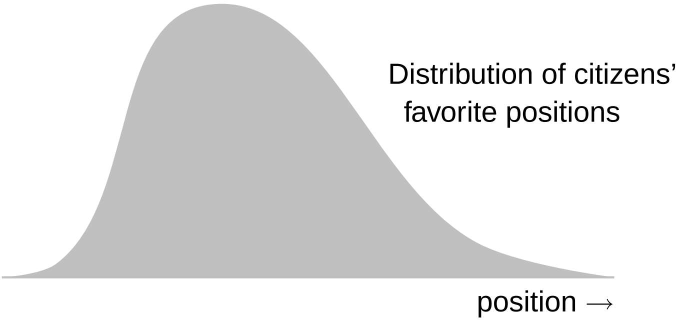
- Political position is number
- Set of positions represents left–right spectrum
- Each citizen has favorite position
- Large number of citizens

position $\longrightarrow$

Electoral competition: Hotelling's model

Model

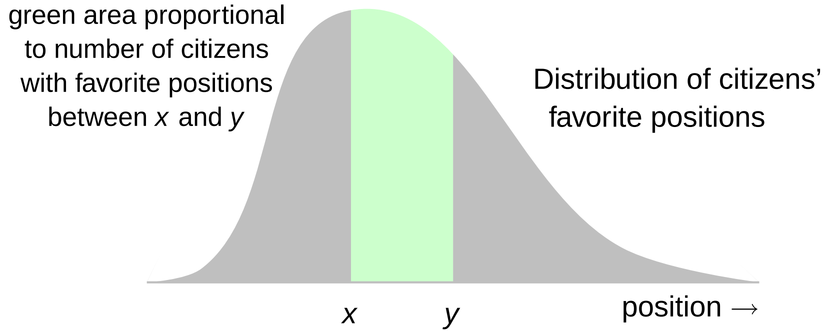
- Political position is number
- Set of positions represents left–right spectrum
- Each citizen has favorite position
- Large number of citizens



Electoral competition: Hotelling's model

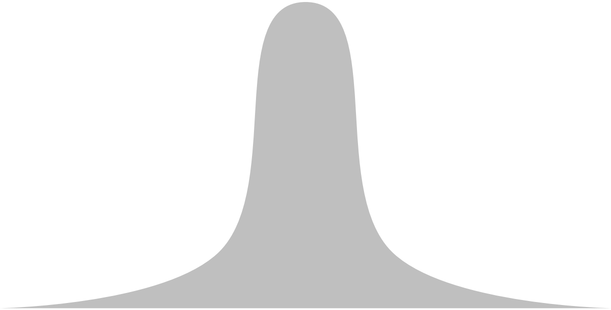
Model

- Political position is number
- Set of positions represents left–right spectrum
- Each citizen has favorite position
- Large number of citizens



Electoral competition: Hotelling's model

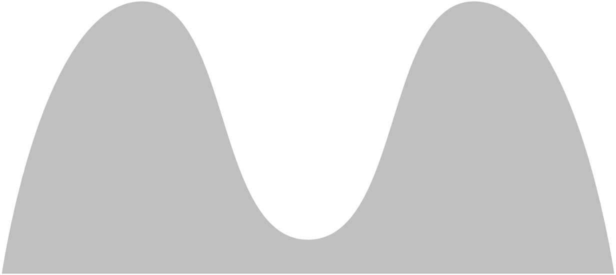
Examples of distributions of citizens' preferences



Few extremists, most citizens favor centrist position

Electoral competition: Hotelling's model

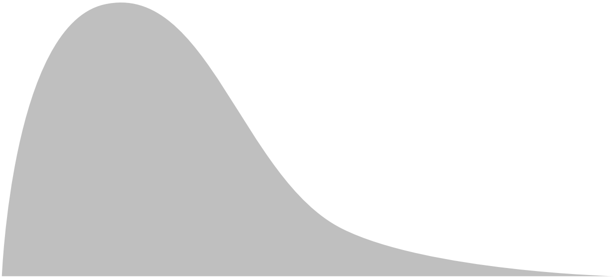
Examples of distributions of citizens' preferences



Many extremists on both sides, few centrists

Electoral competition: Hotelling's model

Examples of distributions of citizens' preferences

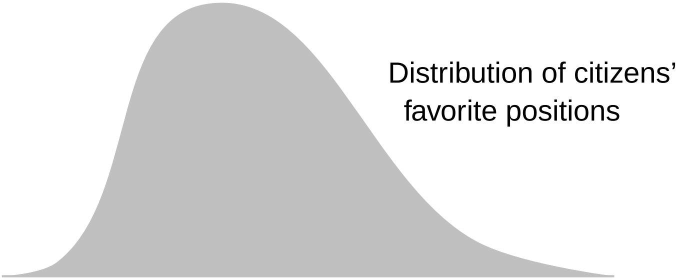


Many citizens with favorite positions on left,
few with favorite positions on right

Electoral competition: Hotelling's model

Model

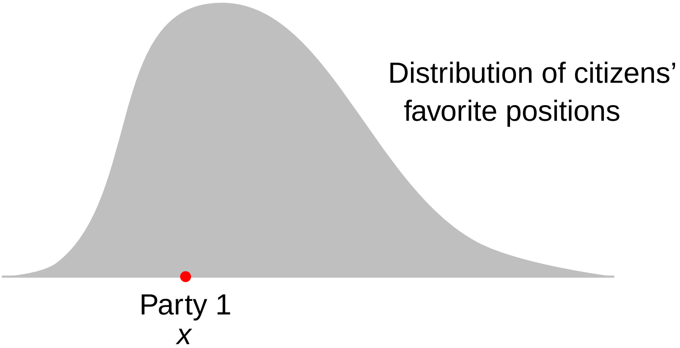
- Each party chooses position



Electoral competition: Hotelling's model

Model

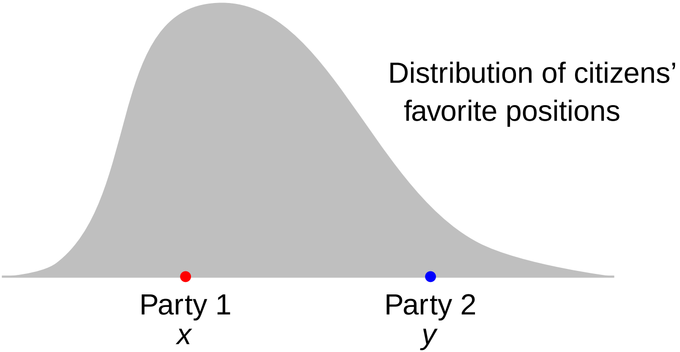
- Each party chooses position



Electoral competition: Hotelling's model

Model

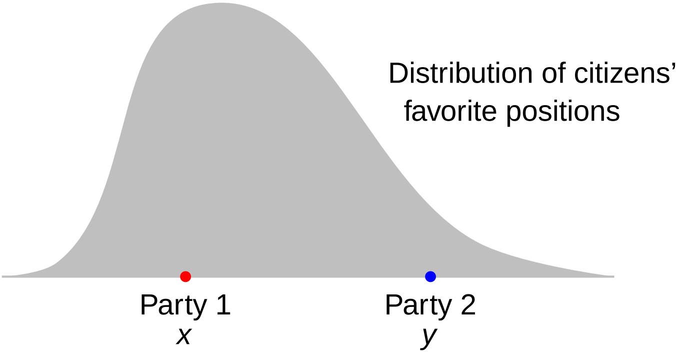
- Each party chooses position



Electoral competition: Hotelling's model

Model

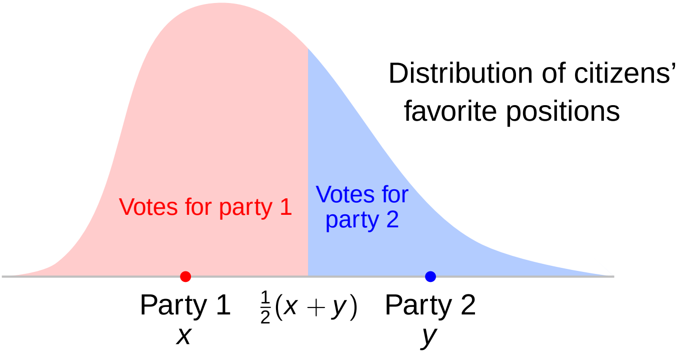
- Each party chooses position
- Each citizen votes for party with position closest to her favorite position—that is, she votes *sincerely*



Electoral competition: Hotelling's model

Model

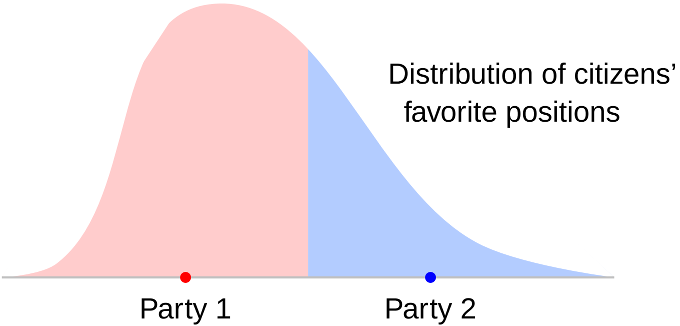
- Each party chooses position
- Each citizen votes for party with position closest to her favorite position—that is, she votes *sincerely*



Electoral competition: Hotelling's model

Model

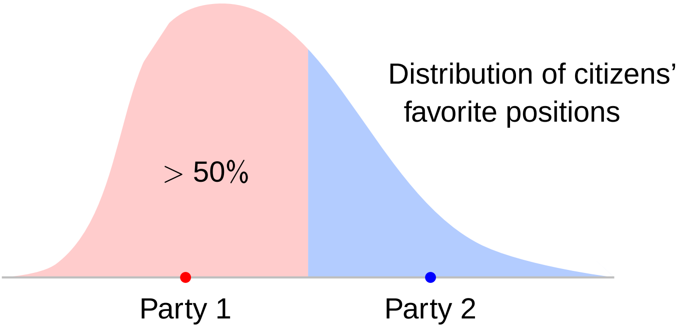
- Party who obtains most votes wins



Electoral competition: Hotelling's model

Model

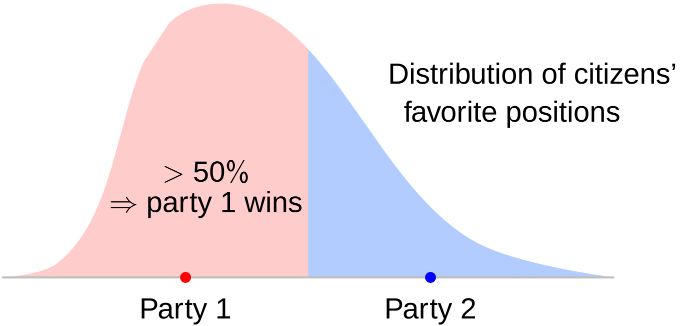
- Party who obtains most votes wins



Electoral competition: Hotelling's model

Model

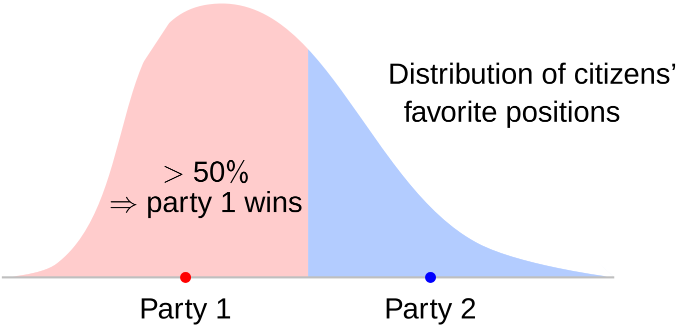
- Party who obtains most votes wins



Electoral competition: Hotelling's model

Model

- Party who obtains most votes wins
- Each party cares only about winning; *no party has ideological attachment to any position*



Electoral competition: Hotelling's model

Strategic game

- Players: parties

Electoral competition: Hotelling's model

Strategic game

- Players: parties
- For each party,
  - possible actions: positions

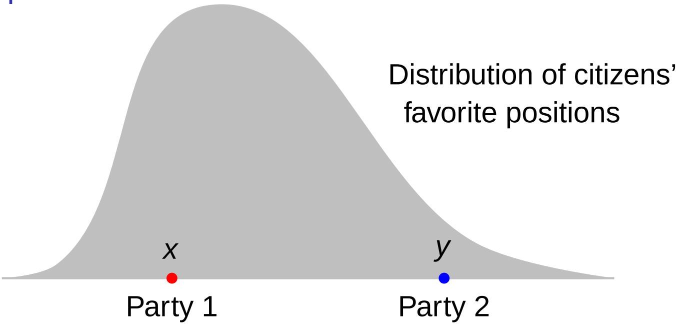
Electoral competition: Hotelling's model

Strategic game

- Players: parties
- For each party,
  - possible actions: positions
  - preferences: win $\succ$ tie $\succ$ lose

Hotelling's model with two parties

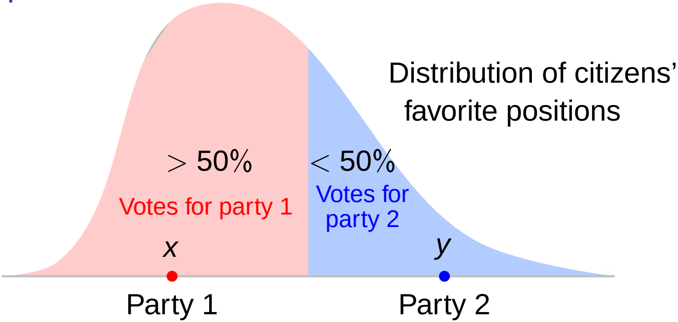
Nash equilibrium



Equilibrium with parties at x and y?

Hotelling's model with two parties

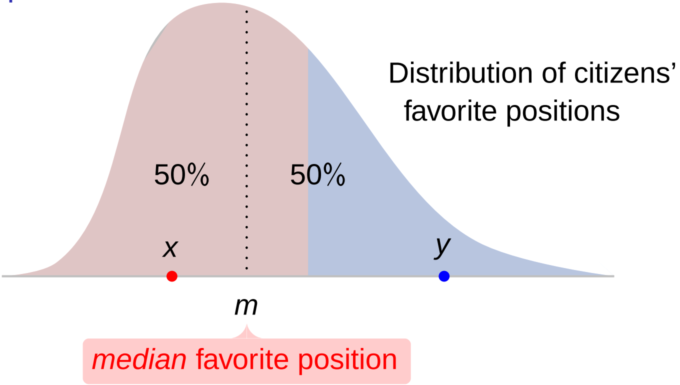
Nash equilibrium



Equilibrium with parties at x and y?

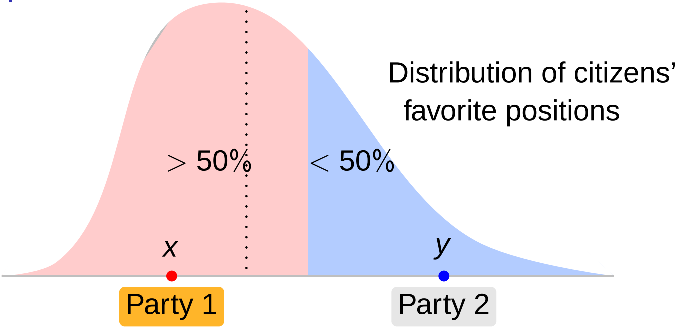
Hotelling's model with two parties

Nash equilibrium



Hotelling's model with two parties

Nash equilibrium

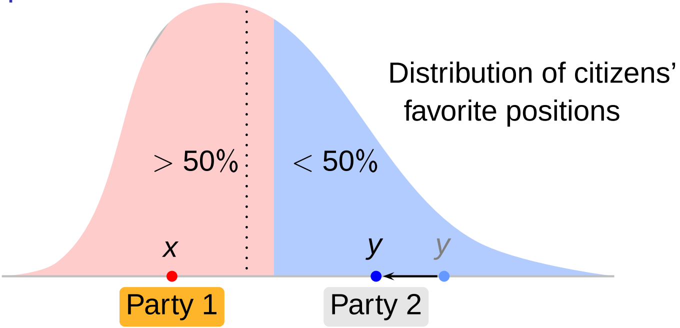


Equilibrium with parties at x and y?

- Party 2 loses

Hotelling's model with two parties

Nash equilibrium

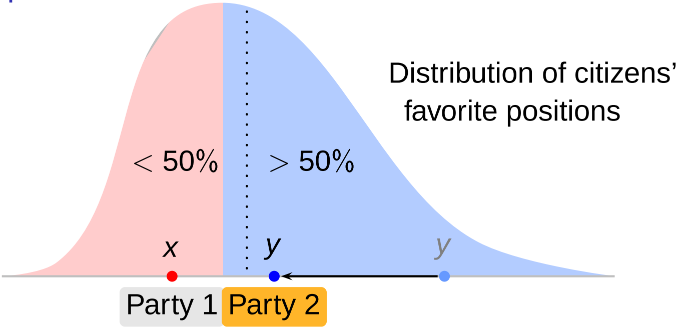


Equilibrium with parties at x and y?

- Party 2 loses
- If party 2 moves left, its vote share increases

Hotelling's model with two parties

Nash equilibrium

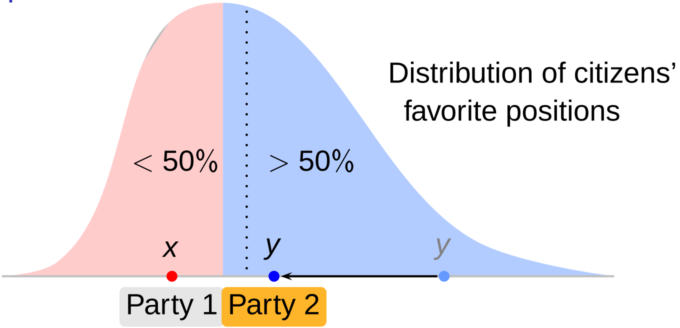


Equilibrium with parties at x and y?

- Party 2 loses
- If party 2 moves left, its vote share increases
- If party 2 moves far enough left, it wins

Hotelling's model with two parties

Nash equilibrium



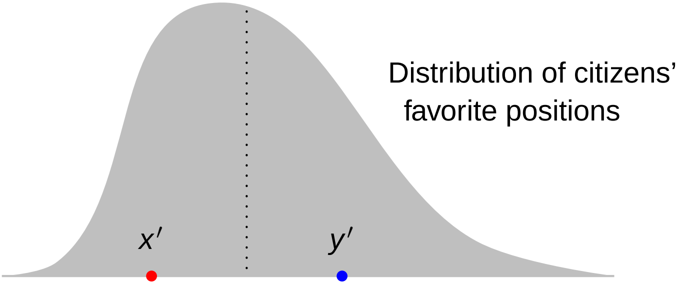
Equilibrium with parties at x and y?

- Party 2 loses
- If party 2 moves left, its vote share increases
- If party 2 moves far enough left, it wins

$\Rightarrow$ not Nash equilibrium

Hotelling's model with two parties

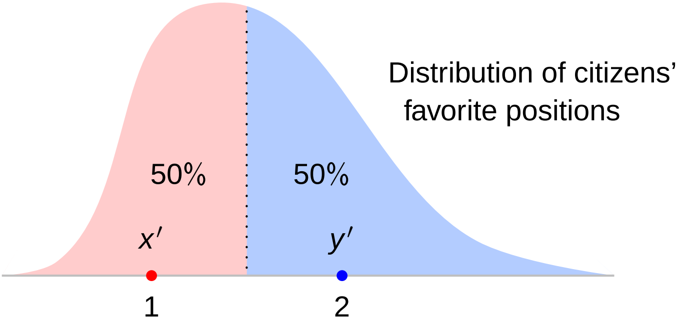
Nash equilibrium



Equilibrium with parties at x' and y'?

Hotelling's model with two parties

Nash equilibrium

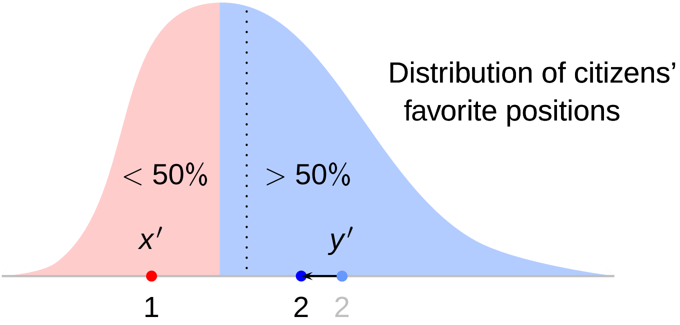


Equilibrium with parties at x' and y'?

- Parties tie

Hotelling's model with two parties

Nash equilibrium

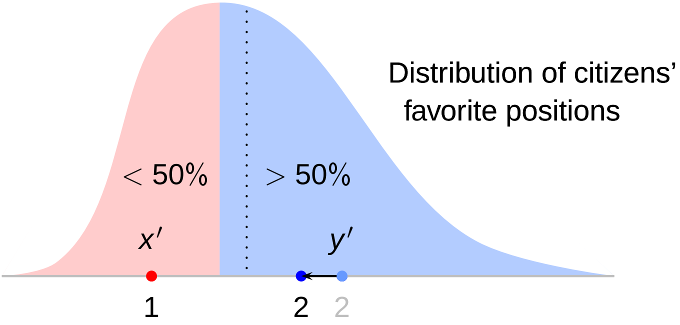


Equilibrium with parties at x' and y'?

- Parties tie
- Party 2 can move slightly left and win

Hotelling's model with two parties

Nash equilibrium



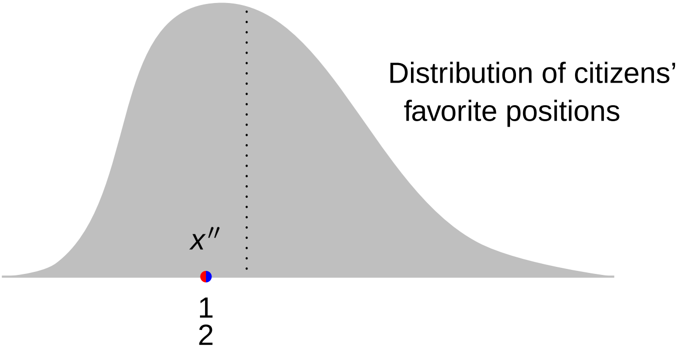
Equilibrium with parties at x' and y'?

- Parties tie
- Party 2 can move slightly left and win

$\Rightarrow$ not Nash equilibrium

Hotelling's model with two parties

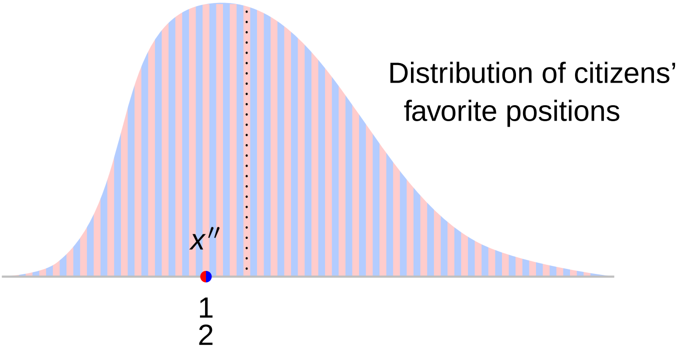
Nash equilibrium



Equilibrium with both parties at x''?

Hotelling's model with two parties

Nash equilibrium

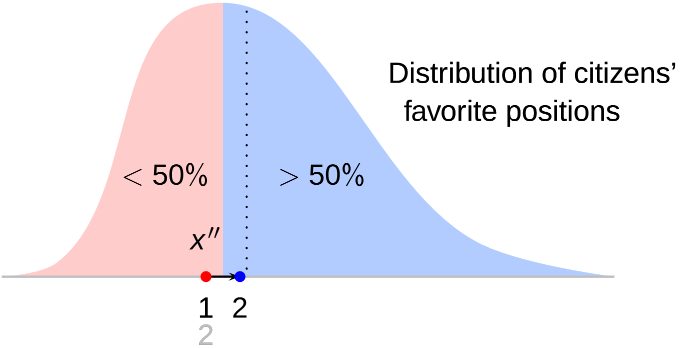


Equilibrium with both parties at x''?

- Parties tie

Hotelling's model with two parties

Nash equilibrium

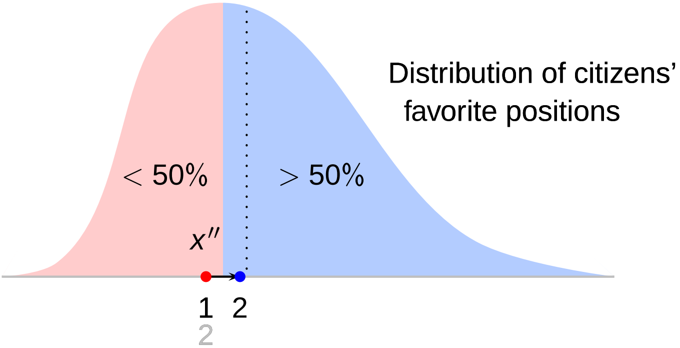


Equilibrium with both parties at x''?

- Parties tie
- Party 2 (for example) can deviate slightly to right and win

Hotelling's model with two parties

Nash equilibrium



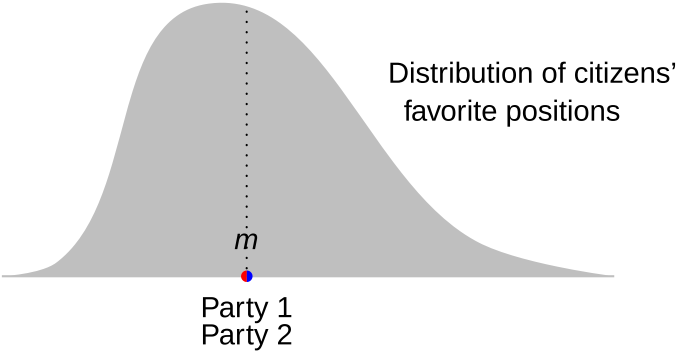
Equilibrium with both parties at x''?

- Parties tie
- Party 2 (for example) can deviate slightly to right and win

$\Rightarrow$ not Nash equilibrium

Hotelling's model with two parties

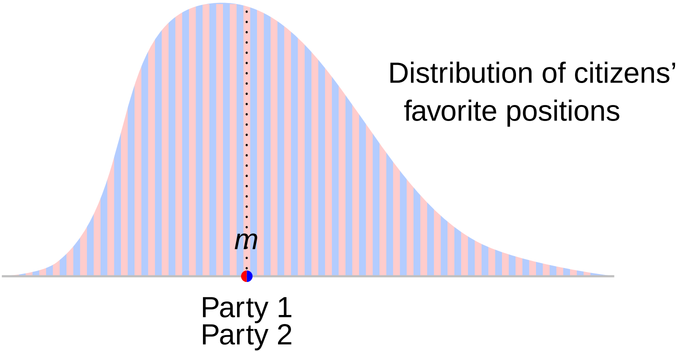
Nash equilibrium



Equilibrium with both parties at m?

Hotelling's model with two parties

Nash equilibrium

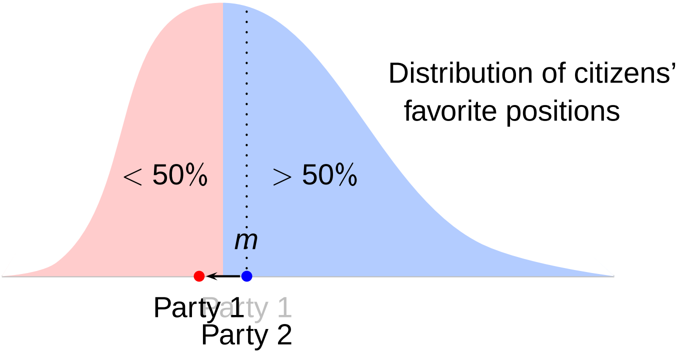


Equilibrium with both parties at *m*?

- Parties tie

Hotelling's model with two parties

Nash equilibrium

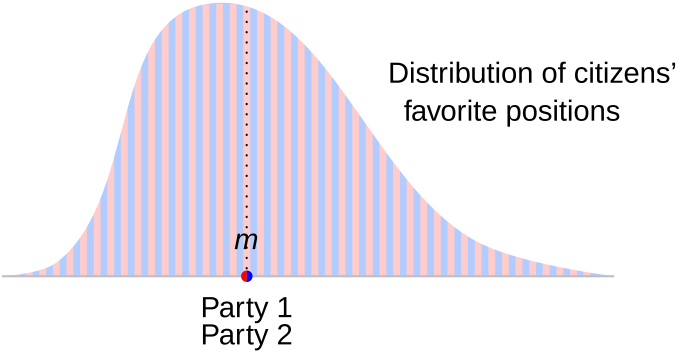


Equilibrium with both parties at m?

- Parties tie
- If either party deviates, it loses

Hotelling's model with two parties

Nash equilibrium



Equilibrium with both parties at m?

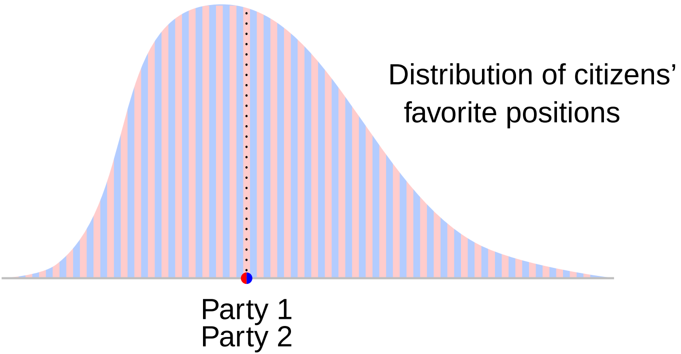
- Parties tie
- If either party deviates, it loses

⇒ Nash equilibrium

Hotelling's model with two parties

Nash equilibrium: conclusion

Unique Nash equilibrium, in which both parties choose *median* favorite position of citizens



Hotelling's model with two parties

Nash equilibrium with two parties: Proof

- *Median* favorite position: position m such that exactly half of citizens' favorite positions are $\leq m$ and half are $\geq m$. (Assume distribution of favorite positions is such that there is only one position with this property.)

Hotelling's model with two parties

Nash equilibrium with two parties: Proof

- *Median* favorite position: position m such that exactly half of citizens' favorite positions are $\leq m$ and half are $\geq m$. (Assume distribution of favorite positions is such that there is only one position with this property.)
- (m, m) is an equilibrium: results in tie;

Hotelling's model with two parties

Nash equilibrium with two parties: Proof

- *Median* favorite position: position m such that exactly half of citizens' favorite positions are $\leq m$ and half are $\geq m$. (Assume distribution of favorite positions is such that there is only one position with this property.)
- (m, m) is an equilibrium: results in tie; if either party chooses position different from m, then it loses

Hotelling's model with two parties

Nash equilibrium with two parties: Proof

- *Median* favorite position: position m such that exactly half of citizens' favorite positions are $\leq m$ and half are $\geq m$. (Assume distribution of favorite positions is such that there is only one position with this property.)
- (m, m) is an equilibrium: results in tie; if either party chooses position different from m, then it loses
- No other pair of positions is a Nash equilibrium:

Hotelling's model with two parties

Nash equilibrium with two parties: Proof

- *Median* favorite position: position m such that exactly half of citizens' favorite positions are $\leq m$ and half are $\geq m$. (Assume distribution of favorite positions is such that there is only one position with this property.)
- (m, m) is an equilibrium: results in tie; if either party chooses position different from m, then it loses
- No other pair of positions is a Nash equilibrium:
  - If one party loses, it can do better by moving to m, where it wins outright if opponent's position $\neq m$ and ties for first place if opponent's position $= m$

Hotelling's model with two parties

Nash equilibrium with two parties: Proof

- *Median* favorite position: position m such that exactly half of citizens' favorite positions are $\leq m$ and half are $\geq m$. (Assume distribution of favorite positions is such that there is only one po
- (m, m) is an e ... y chooses posit
- No other pair
  - If one party loses, it can do better by moving to m, where it wins outright if opponent's position $\neq m$ and ties for first place if opponent's position $= m$

This deviation differs from one in argument on a previous slide. Both are valid; one here makes argument more compact.

Hotelling's model with two parties

Nash equilibrium with two parties: Proof

- *Median* favorite position: position m such that exactly half of citizens' favorite positions are $\leq m$ and half are $\geq m$. (Assume distribution of favorite positions is such that there is only one position with this property.)
- (m, m) is an equilibrium: results in tie; if either party chooses position different from m, then it loses
- No other pair of positions is a Nash equilibrium:
  - If one party loses, it can do better by moving to m, where it wins outright if opponent's position $\neq m$ and ties for first place if opponent's position $= m$
  - If parties tie (because their positions are either the same or symmetric about m), either party can do better by moving to m, where it wins outright

Hotelling's model with two parties

- Parties don't generally adopt same position

Hotelling's model with two parties

- Parties don't generally adopt same position
- What ingredient is missing from model?

Hotelling's model with two parties

- Parties don't generally adopt same position
- What ingredient is missing from model?
- Parties should care about position, not only about winning?

Hotelling's model with two parties

- Parties don't generally adopt same position
- What ingredient is missing from model?
- Parties should care about position, not only about winning?
- Consider case in which each party cares *only* about the *position of the winning party*

Hotelling's model with two parties

- Parties don't generally adopt same position
- What ingredient is missing from model?
- Parties should care about position, not only about winning?
- Consider case in which each party cares *only* about the *position of the winning party*
- Assume that if parties tie for votes, policy is average of parties' positions

Parties that care about winning position

Strategic game

- Players: two parties

Parties that care about winning position

Strategic game

- Players: two parties
- For each party i,
  - possible actions: positions

Parties that care about winning position

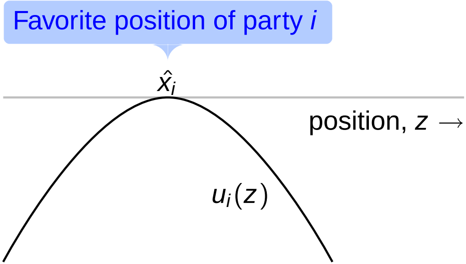
Strategic game

- Players: two parties
- For each party i,
  - possible actions: positions
  - payoff:

Parties that care about winning position

Strategic game

- Players: two parties
- For each party i,
  - possible actions: positions
  - payoff:



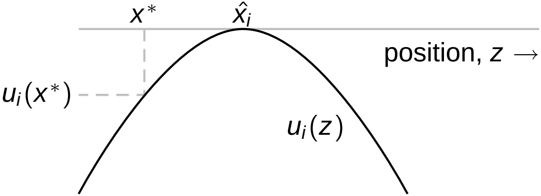
Payoff of party i, with favorite position $\hat{x}_i$

Parties that care about winning position

Strategic game

- Players: two parties
- For each party i,
  - possible actions: positions
  - payoff:

i's payoff when policy of winner is x^*

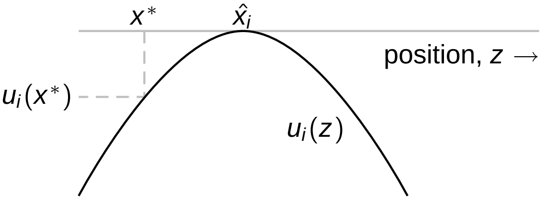


Payoff of party i, with favorite position $\hat{x}_i$

Parties that care about winning position

Strategic game

- Players: two parties
- For each party i,
  - possible actions: positions
  - payoff: $u_i(x^*)$, where x^* is position of winner (or average of winners' positions if tied) and u_i has single peak, at $\hat{x}_i$

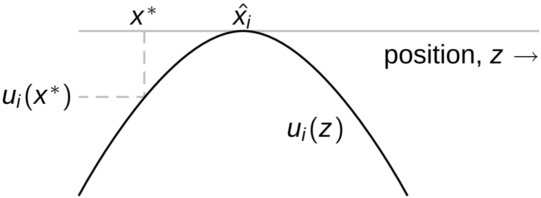


Payoff of party i, with favorite position $\hat{x}_i$

Parties that care about winning position

Strategic game

- Players: two parties
- For each party i,
  - possible actions: positions
  - payoff: $u_i(x^*)$, where x^* is position of winner (or average of winners' positions if tied) and u_i has single peak, at $\hat{x}_i$

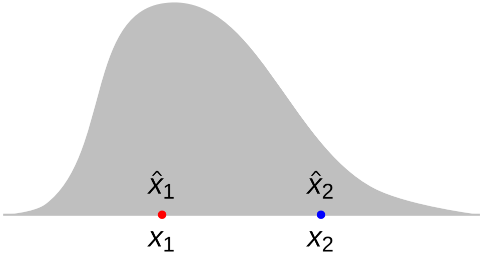


Payoff of party i, with favorite position $\hat{x}_i$

Assume $\hat{x}_1 < m < \hat{x}_2$ (one party on left and one on right)

Parties that care about winning position

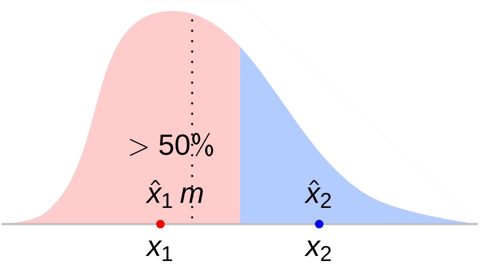
Nash equilibrium



Equilibrium in which each party chooses its favorite position?

Parties that care about winning position

Nash equilibrium

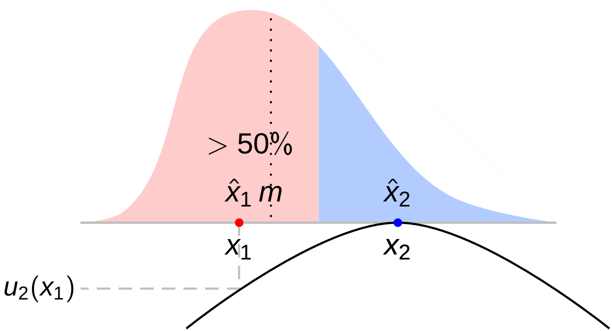


Equilibrium in which each party chooses its favorite position?

- Suppose positions such that party 1 wins

Parties that care about winning position

Nash equilibrium



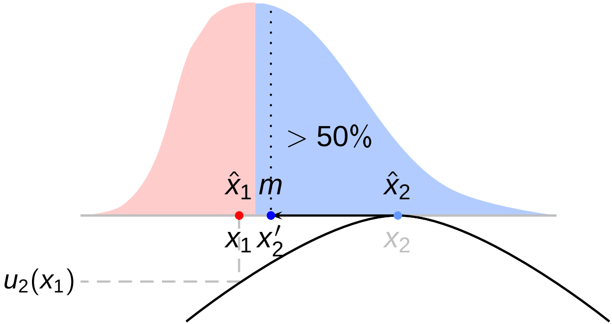
Equilibrium in which each party chooses its favorite position?

- Suppose positions such that party 1 wins

$\Rightarrow$ party 2's payoff $u_2(x_1)$

Parties that care about winning position

Nash equilibrium



Equilibrium in which each party chooses its favorite position?

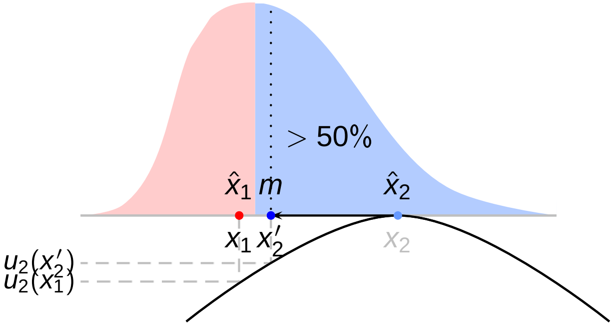
- Suppose positions such that party 1 wins

⇒ party 2's payoff $u_2(x_1)$

- Party 2 moves to m

Parties that care about winning position

Nash equilibrium



Equilibrium in which each party chooses its favorite position?

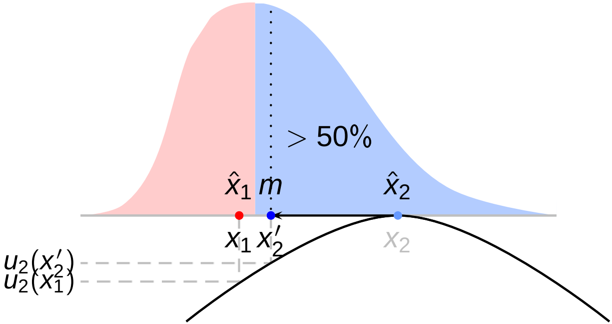
- Suppose positions such that party 1 wins

$\Rightarrow$ party 2's payoff $u_2(x_1)$

- Party 2 moves to $m \Rightarrow$ wins and gets $u_2(x_2') > u_2(x_1)$

Parties that care about winning position

Nash equilibrium



Equilibrium in which each party chooses its favorite position?

- Suppose positions such that party 1 wins

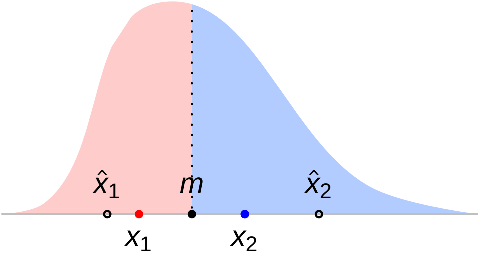
$\Rightarrow$ party 2's payoff $u_2(x_1)$

- Party 2 moves to m $\Rightarrow$ wins and gets $u_2(x_2') > u_2(x_1)$

$\Rightarrow$ not Nash equilibrium

Parties that care about winning position

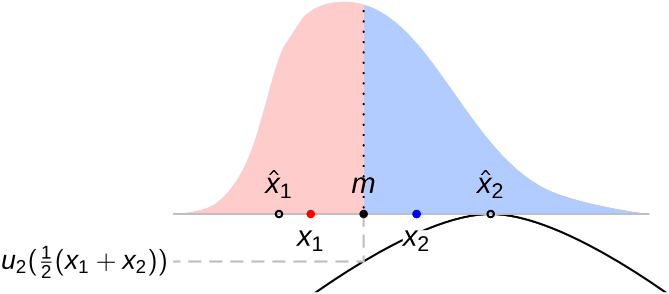
Nash equilibrium



Equilibrium in which parties tie and moderate their positions?

Parties that care about winning position

Nash equilibrium

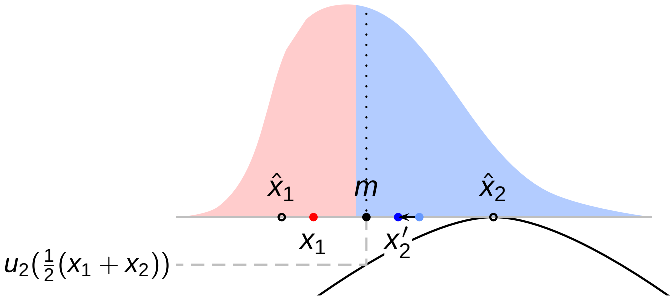


Equilibrium in which parties tie and moderate their positions?

- Outcome is $\frac{1}{2}(x_1 + x_2) = m$

Parties that care about winning position

Nash equilibrium

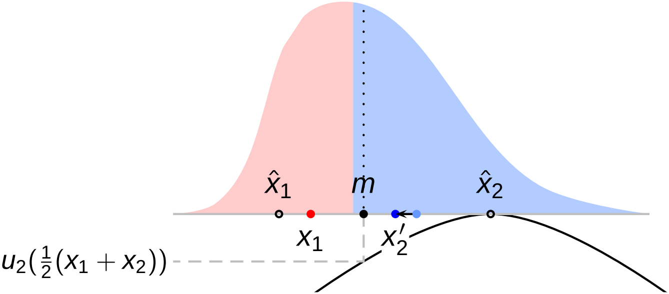


Equilibrium in which parties tie and moderate their positions?

- Outcome is $\frac{1}{2}(x_1 + x_2) = m \Rightarrow$ party 2's payoff $u_2(\frac{1}{2}(x_1 + x_2)) = u_2(m)$

Parties that care about winning position

Nash equilibrium

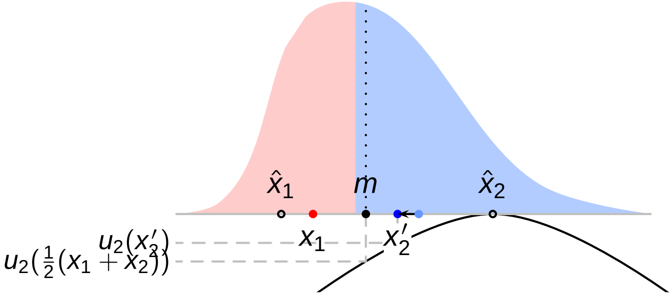


Equilibrium in which parties tie and moderate their positions?

- Outcome is $\frac{1}{2}(x_1 + x_2) = m \Rightarrow$ party 2's payoff $u_2(\frac{1}{2}(x_1 + x_2)) = u_2(m)$
- Party 2 moves to left $\Rightarrow$ party 2 wins

Parties that care about winning position

Nash equilibrium

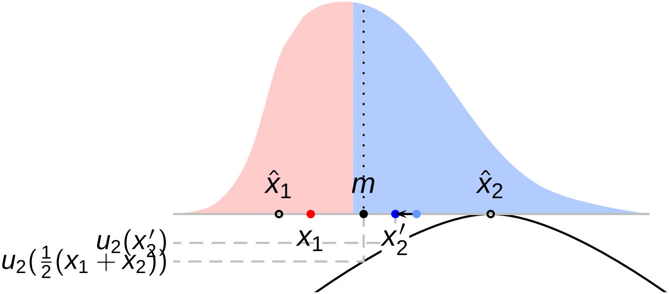


Equilibrium in which parties tie and moderate their positions?

- Outcome is $\frac{1}{2}(x_1 + x_2) = m \Rightarrow$ party 2's payoff $u_2(\frac{1}{2}(x_1 + x_2)) = u_2(m)$
- Party 2 moves to left $\Rightarrow$ party 2 wins and gets payoff $u_2(x_2') > u_2(\frac{1}{2}(x_1 + x_2)) = u_2(m)$

Parties that care about winning position

Nash equilibrium



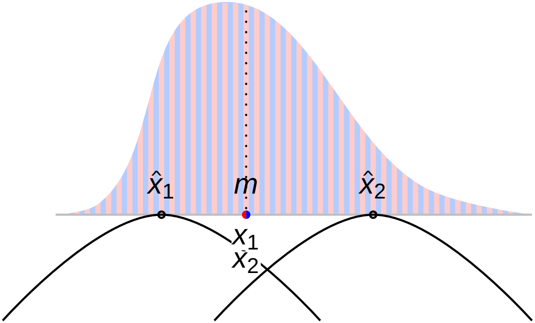
Equilibrium in which parties tie and moderate their positions?

- Outcome is $\frac{1}{2}(x_1 + x_2) = m \Rightarrow$ party 2's payoff $u_2(\frac{1}{2}(x_1 + x_2)) = u_2(m)$
- Party 2 moves to left $\Rightarrow$ party 2 wins and gets payoff $u_2(x_2') > u_2(\frac{1}{2}(x_1 + x_2)) = u_2(m)$

$\Rightarrow$ not Nash equilibrium

Parties that care about winning position

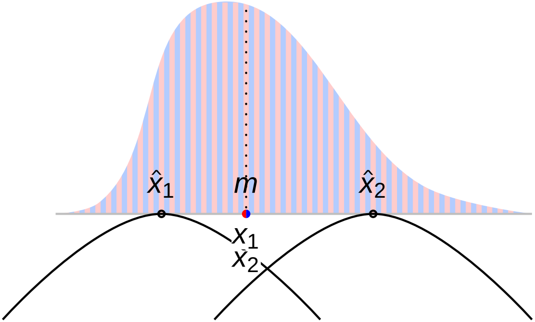
Nash equilibrium



Equilibrium in which parties both choose median position?

Parties that care about winning position

Nash equilibrium

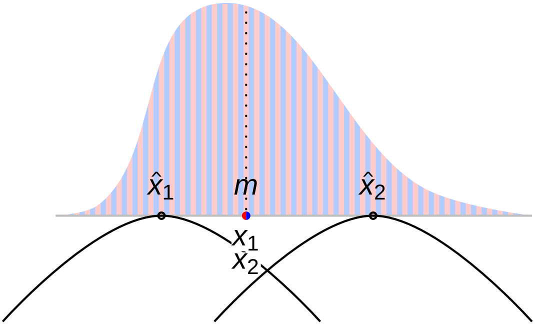


Equilibrium in which parties both choose median position?

- Outcome is $m \Rightarrow$ party i's payoff $u_i(m)$

Parties that care about winning position

Nash equilibrium

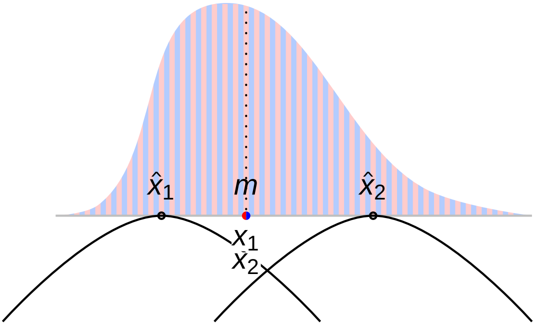


Equilibrium in which parties both choose median position?

- Outcome is $m \Rightarrow$ party i's payoff $u_i(m)$
- Either party moves $\Rightarrow$ loses

Parties that care about winning position

Nash equilibrium

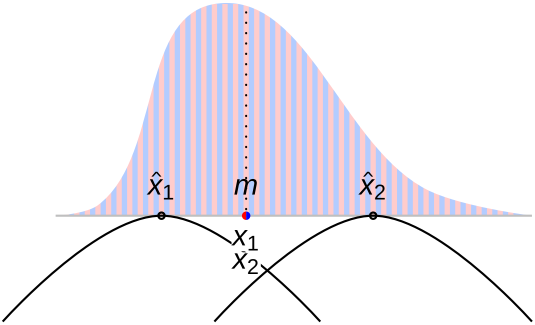


Equilibrium in which parties both choose median position?

- Outcome is $m \Rightarrow$ party i's payoff $u_i(m)$
- Either party moves $\Rightarrow$ loses $\Rightarrow$ outcome unchanged

Parties that care about winning position

Nash equilibrium



Equilibrium in which parties both choose median position?

- Outcome is $m \Rightarrow$ party i's payoff $u_i(m)$
- Either party moves $\Rightarrow$ loses $\Rightarrow$ outcome unchanged

$\Rightarrow$ Nash equilibrium!

Parties that care about winning position

If we check all possible configurations of positions we find . . .

Parties that care about winning position

If we check all possible configurations of positions we find ...

Nash equilibrium

Parties care only about winning position $\Rightarrow$ game has unique Nash equilibrium, in which both parties choose median of citizens' favorite positions

Parties that care about winning position

If we check all possible configurations of positions we find . . .

Nash equilibrium

Parties care only about winning position $\Rightarrow$ game has unique Nash equilibrium, in which both parties choose median of citizens' favorite positions

That is: Nash equilibrium outcome is same as in case in which parties care only about winning!

Parties that care about winning position

- Intuitively, when party moves closer to its rival it faces tradeoff:

Parties that care about winning position

- Intuitively, when party moves closer to its rival it faces tradeoff:
  - higher probability of winning

Parties that care about winning position

- Intuitively, when party moves closer to its rival it faces tradeoff:
  - higher probability of winning
  - less desirable position if wins

Parties that care about winning position

- Intuitively, when party moves closer to its rival it faces tradeoff:
  - higher probability of winning
  - less desirable position if wins
- Model doesn't capture tradeoff because probability of winning is either 1 or 0 (or parties tie)

Parties that care about winning position

- Intuitively, when party moves closer to its rival it faces tradeoff:
  - higher probability of winning
  - less desirable position if wins
- Model doesn't capture tradeoff because probability of winning is either 1 or 0 (or parties tie)
- To capture tradeoff, need to add uncertainty (e.g. about citizens' preferences)

Parties that care about winning position

- Intuitively, when party moves closer to its rival it faces tradeoff:
  - higher probability of winning
  - less desirable position if wins
- Model doesn't capture tradeoff because probability of winning is either 1 or 0 (or parties tie)
- To capture tradeoff, need to add uncertainty (e.g. about citizens' preferences)
- If do so, model becomes difficult to work with, but can yield equilibrium with distinct positions for parties

Hotelling's model: three parties

Three parties

- Return to model in which parties care only about winning, and consider case of *three* parties

Hotelling's model: three parties

Three parties

- Return to model in which parties care only about winning, and consider case of *three* parties
- Suppose each party has option of staying out (quitting), which is better than losing (and worse than tying)

Hotelling's model: three parties

Three parties

- Return to model in which parties care only about winning, and consider case of *three* parties
- Suppose each party has option of staying out (quitting), which is better than losing (and worse than tying)

Strategic game

- Players: three parties
- For each party,
  - possible actions: $\{Out\} \cup$ set of possible positions
  - preferences: win $\succ$ tie $\succ$ *Out* $\succ$ lose

Hotelling's model: three parties

Three parties: Nash equilibrium

Claim In Nash equilibrium, a party that runs either wins outright or ties for first place—it does not lose

Hotelling's model: three parties

Three parties: Nash equilibrium

Claim In Nash equilibrium, a party that runs either wins outright or ties for first place—it does not lose

Argument Party that runs and loses can quit, which it prefers to losing

Hotelling's model: three parties

Three parties: Nash equilibrium

Claim In Nash equilibrium, a party that runs either wins outright or ties for first place—it does not lose

Argument Party that runs and loses can quit, which it prefers to losing

Now consider possible configurations:

- **No party enters**

Hotelling's model: three parties

Three parties: Nash equilibrium

Claim In Nash equilibrium, a party that runs either wins outright or ties for first place—it does not lose

Argument Party that runs and loses can quit, which it prefers to losing

Now consider possible configurations:

- **No party enters** Not equilibrium: a party can deviate and enter, and win

Hotelling's model: three parties

Three parties: Nash equilibrium

Claim In Nash equilibrium, a party that runs either wins outright or ties for first place—it does not lose

Argument Party that runs and loses can quit, which it prefers to losing

Now consider possible configurations:

- **No party enters** Not equilibrium: a party can deviate and enter, and win
- **One party enters**

Hotelling's model: three parties

Three parties: Nash equilibrium

Claim In Nash equilibrium, a party that runs either wins outright or ties for first place—it does not lose

Argument Party that runs and loses can quit, which it prefers to losing

Now consider possible configurations:

- **No party enters** Not equilibrium: a party can deviate and enter, and win
- **One party enters** Not equilibrium: another party can enter at same position and tie for first place

Hotelling's model: three parties

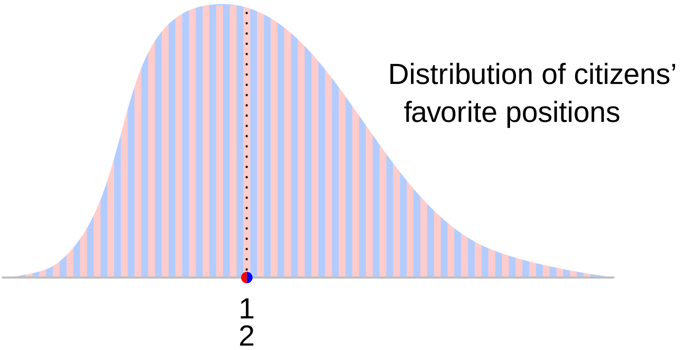
Three parties: Nash equilibrium

- **Two parties enter**

Hotelling's model: three parties

Three parties: Nash equilibrium

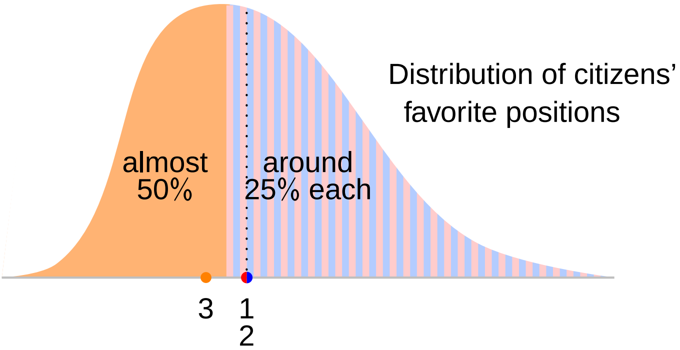
- **Two parties enter**
  - Must both choose median (by argument in two-party game)



Hotelling's model: three parties

Three parties: Nash equilibrium

- **Two parties enter**
  - Must both choose median (by argument in two-party game)
  - But then third party can enter near median and win—so not Nash equilibrium



Hotelling's model: three parties

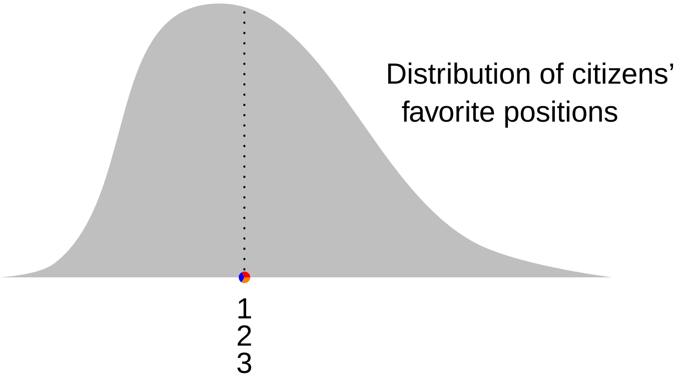
Three parties: Nash equilibrium

- **Three parties enter**

Hotelling's model: three parties

Three parties: Nash equilibrium

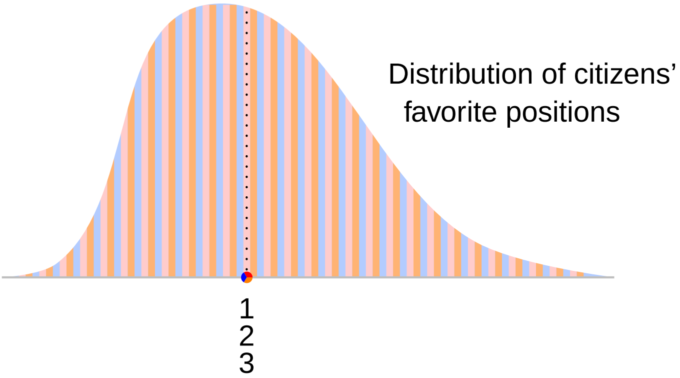
- **Three parties enter**
  - all choose median



Hotelling's model: three parties

Three parties: Nash equilibrium

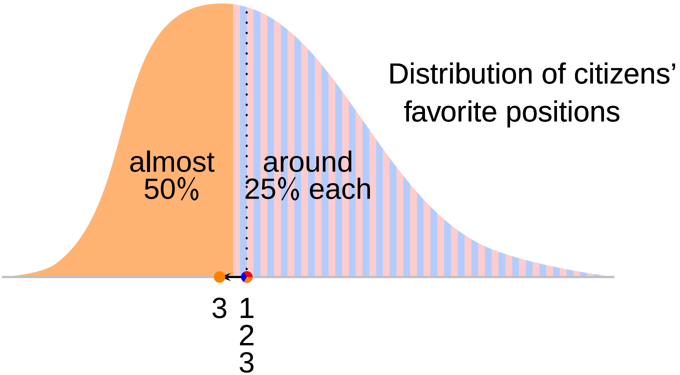
- **Three parties enter**
  - all choose median ⇒ they tie



Hotelling's model: three parties

Three parties: Nash equilibrium

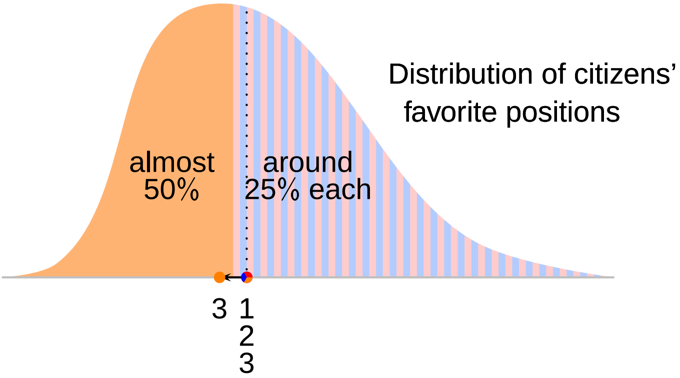
- **Three parties enter**
  - all choose median ⇒ they tie
  - one party deviates a little ⇒ it wins



Hotelling's model: three parties

Three parties: Nash equilibrium

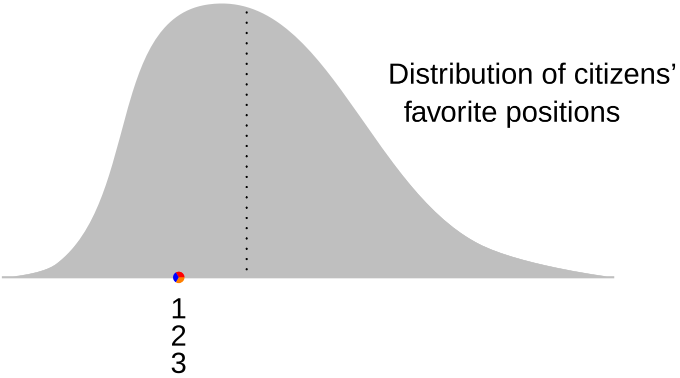
- **Three parties enter**
  - all choose median ⇒ they tie
  - one party deviates a little ⇒ it wins
  - so not Nash equilibrium



Hotelling's model: three parties

Three parties: Nash equilibrium

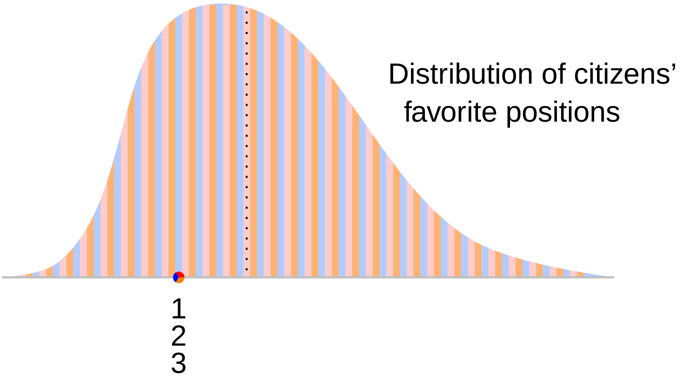
- **Three parties enter**
  - all choose same position, $\neq$ median



Hotelling's model: three parties

Three parties: Nash equilibrium

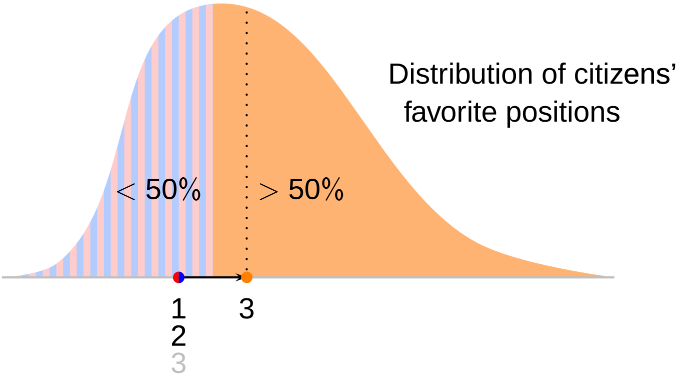
- **Three parties enter**
  - all choose same position, $\neq$ median $\Rightarrow$ they tie



Hotelling's model: three parties

Three parties: Nash equilibrium

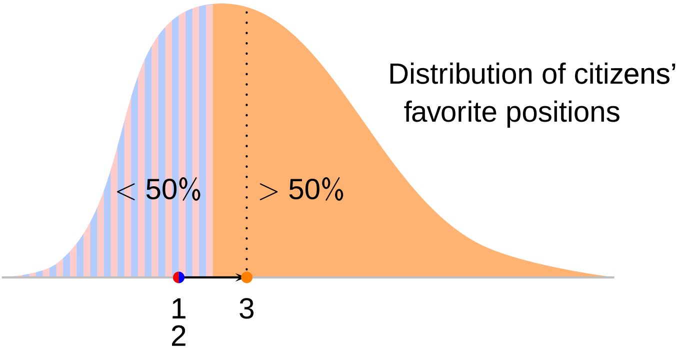
- **Three parties enter**
  - all choose same position, $\neq$ median $\Rightarrow$ they tie
  - one party deviates to median $\Rightarrow$ it wins



Hotelling's model: three parties

Three parties: Nash equilibrium

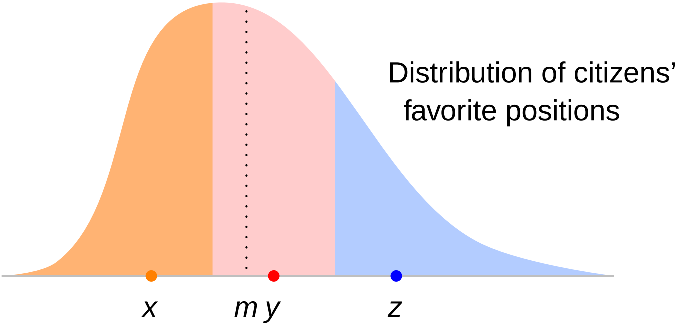
- **Three parties enter**
  - all choose same position, $\neq$ median $\Rightarrow$ they tie
  - one party deviates to median $\Rightarrow$ it wins
  - so not Nash equilibrium



Hotelling's model: three parties

Three parties: Nash equilibrium

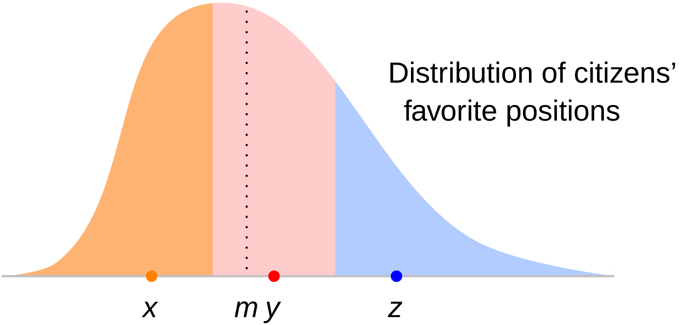
- **Three parties enter**
  - choose different positions



Hotelling's model: three parties

Three parties: Nash equilibrium

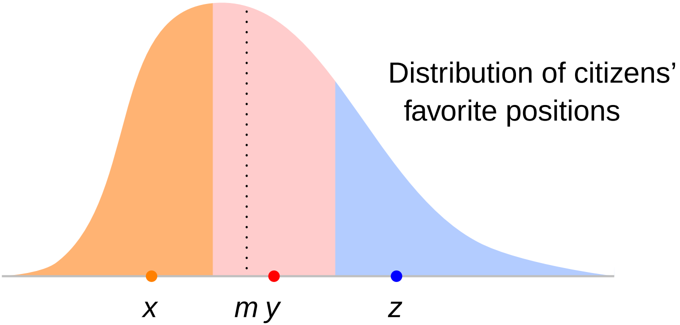
- **Three parties enter**
  - choose different positions $\Rightarrow$ must tie (else would exit)



Hotelling's model: three parties

Three parties: Nash equilibrium

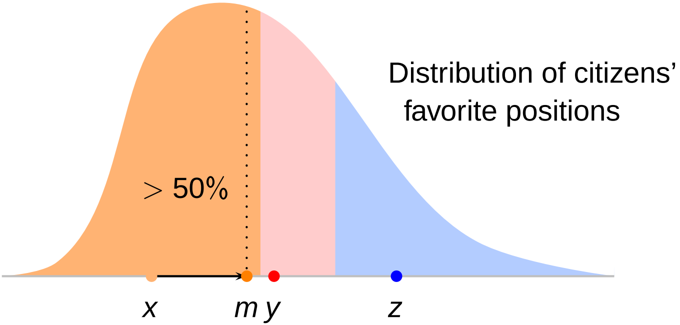
- **Three parties enter**
  - choose different positions $\Rightarrow$ must tie (else would exit)
  - Suppose positions $x < m < y < z$



Hotelling's model: three parties

Three parties: Nash equilibrium

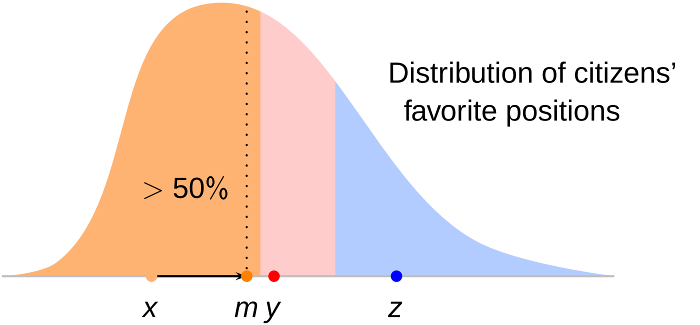
- **Three parties enter**
  - choose different positions ⇒ must tie (else would exit)
  - Suppose positions $x < m < y < z$ ⇒ party at x can move to m and win outright



Hotelling's model: three parties

Three parties: Nash equilibrium

- **Three parties enter**
  - choose different positions ⇒ must tie (else would exit)
  - Suppose positions $x < m < y < z$ ⇒ party at x can move to m and win outright
  - Not Nash equilibrium



Hotelling's model: three parties

Three parties: Nash equilibrium

- **Three parties enter**
  - choose different positions $\Rightarrow$ must tie (else would exit)
    - $x < y < m < z \Rightarrow$ party at z can move to m and win outright
    - $x < y = m < z \Rightarrow$ party at x can move close to m and win outright
    - $x = y < m < z \Rightarrow$ party at z can move to m and win outright
    - $x = y = m < z \Rightarrow$ party at z can move close to m and win outright
    - Note that $x < y < z \leq m$ is not possible, because party at z then wins outright

Conclusion

The game has no Nash equilibrium!

Summary

- Two parties whose only objective is to win $\Rightarrow$ both choose median of citizens' favorite positions

Summary

- Two parties whose only objective is to win $\Rightarrow$ both choose median of citizens' favorite positions
- Two "ideological" parties, who care only about position of winner $\Rightarrow$ both choose median of citizens' favorite positions

Summary

- Two parties whose only objective is to win $\Rightarrow$ both choose median of citizens' favorite positions
- Two "ideological" parties, who care only about position of winner $\Rightarrow$ both choose median of citizens' favorite positions
- Three parties whose only objective is to win $\Rightarrow$ no Nash equilibrium!

Summary

- Two parties whose only objective is to win ⇒ both choose median of citizens' favorite positions
- Two "ideological" parties, who care only about position of winner ⇒ both choose median of citizens' favorite positions
- Three parties whose only objective is to win ⇒ no Nash equilibrium!
- So no model so far consistent with two parties at different positions, or with three parties

Citizen-candidates

Model

- Each citizen decides whether to become a candidate

Citizen-candidates

Model

- Each citizen decides whether to become a candidate
- Decisions are made simultaneously

Citizen-candidates

Model

- Each citizen decides whether to become a candidate
- Decisions are made simultaneously
- *Candidates cannot hide their true preferences*; i's favorite position is $\hat{x}_i$

Citizen-candidates

Model

- Each citizen decides whether to become a candidate
- Decisions are made simultaneously
- *Candidates cannot hide their true preferences*; i's favorite position is $\hat{x}_i$
- Running as a candidate entails a cost $c > 0$

Citizen-candidates

Model

- Each citizen decides whether to become a candidate
- Decisions are made simultaneously
- *Candidates cannot hide their true preferences*; i's favorite position is $\hat{x}_i$
- Running as a candidate entails a cost $c > 0$
- After entry decisions are made, citizens vote sincerely

Citizen-candidates

Model

- Each citizen decides whether to become a candidate
- Decisions are made simultaneously
- *Candidates cannot hide their true preferences*; i's favorite position is $\hat{x}_i$
- Running as a candidate entails a cost $c > 0$
- After entry decisions are made, citizens vote sincerely
- If candidates tie for first place, winner is selected randomly (with equal probabilities)

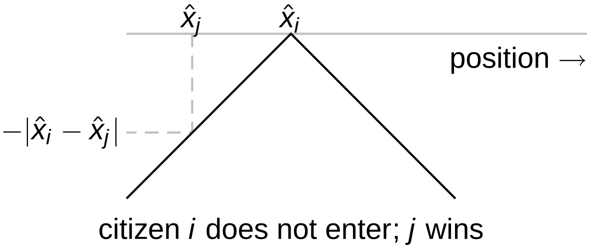
Citizen-candidates

Model

- Each citizen decides whether to become a candidate
- Decisions are made simultaneously
- *Candidates cannot hide their true preferences*; i's favorite position is $\hat{x}_i$
- Running as a candidate entails a cost $c > 0$
- After entry decisions are made, citizens vote sincerely
- If candidates tie for first place, winner is selected randomly (with equal probabilities)
- Winner gets payoff $b > 0$ (in addition to payoff from winning position)

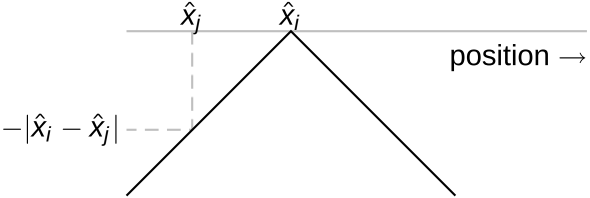
Citizen-candidates

Payoff of citizen i

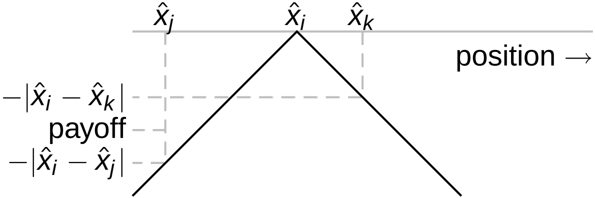


Citizen-candidates

Payoff of citizen i



citizen i does not enter; j wins



citizen i does not enter; j and k tie for most votes

Citizen-candidates

Strategic game

- Players: citizens

Citizen-candidates

Strategic game

- Players: citizens
- For each citizen i,

Citizen-candidates

Strategic game

- Players: citizens
- For each citizen i,
  - possible actions: $\{Run, Out\}$

Citizen-candidates

Strategic game

- Players: citizens
- For each citizen i,
  - possible actions: $\{Run, Out\}$
  - payoff:

$$\begin{cases} \\ \\ \end{cases}$$

Citizen-candidates

Strategic game

- Players: citizens
- For each citizen i,
  - possible actions: $\{Run, Out\}$
  - payoff:

$$\begin{cases} -|\hat{x}_i - \hat{x}_j| & \text{if } i \text{ chooses } Out \text{ and } j \text{ wins} \\ & \\ & \end{cases}$$

Citizen-candidates

Strategic game

- Players: citizens
- For each citizen i,

$Out\}$

negative of distance from *i*'s favorite position to *j*'s favorite position

$$\begin{cases} -|\hat{x}_i - \hat{x}_j| & \text{if } i \text{ chooses } Out \text{ and } j \text{ wins} \end{cases}$$

Citizen-candidates

Strategic game

- Players: citizens
- For each citizen i,
  - possible actions: $\{Run, Out\}$
  - payoff:

$$\begin{cases} -|\hat{x}_i - \hat{x}_j| & \text{if } i \text{ chooses } Out \text{ and } j \text{ wins} \\ -|\hat{x}_i - \hat{x}_j| - c & \text{if } i \text{ chooses } Run \text{ and } j \text{ wins} \\ & \\ & \end{cases}$$

Citizen-candidates

Strategic game

- Players: citizens
- For each citizen i,
  - possible actions: $\{Run, Out\}$
  - payoff:

cost of running as a candidate

$$\begin{cases} -|\hat{x}_i - \hat{x}_j| & \text{if } i \text{ chooses } Out \text{ and } j \text{ wins} \\ -|\hat{x}_i - \hat{x}_j| - c & \text{if } i \text{ chooses } Run \text{ and } j \text{ wins} \\ & \\ & \end{cases}$$

Citizen-candidates

Strategic game

- Players: citizens
- For each citizen i,
  - possible actions: $\{Run, Out\}$
  - payoff:

$$\begin{cases} -|\hat{x}_i - \hat{x}_j| & \text{if } i \text{ chooses } Out \text{ and } j \text{ wins} \\ -|\hat{x}_i - \hat{x}_j| - c & \text{if } i \text{ chooses } Run \text{ and } j \text{ wins} \\ b - c & \text{if } i \text{ chooses } Run \text{ and } i \text{ wins} \end{cases}$$

Citizen-candidates

Strategic game

- Players: citizens
- For each citizen i,
  - possible actions: $\{Run, Out\}$
  - payoff:

$$\begin{cases} -|\hat{x}_i - \hat{x}_j| & \text{if } i \text{ chooses } Out \text{ and } j \text{ wins} \\ -|\hat{x}_i - \hat{x}_j| - c & \text{if } i \text{ chooses } Run \text{ and } j \text{ wins} \\ b - c & \text{if } i \text{ chooses } Run \text{ and } i \text{ wins} \end{cases}$$

direct benefit of winning

Citizen-candidates

Strategic game

- Players: citizens
- For each citizen i,
  - possible actions: $\{Run, Out\}$
  - payoff:

$$\begin{cases} -|\hat{x}_i - \hat{x}_j| & \text{if } i \text{ chooses } Out \text{ and } j \text{ wins} \\ -|\hat{x}_i - \hat{x}_j| - c & \text{if } i \text{ chooses } Run \text{ and } j \text{ wins} \\ b - c & \text{if } i \text{ chooses } Run \text{ and } i \text{ wins} \\ \frac{1}{2}b - \frac{1}{2}|\hat{x}_i - \hat{x}_j| - c & \text{if } i \text{ chooses } Run \text{ and } i \text{ and } j \text{ tie for first place} \end{cases}$$

Citizen-candidates

Strategic game

- Players: citizens
- For each citizen i,
  - possible actions: $\{Run, Out\}$
  - payoff:

i wins with prob. $\frac{1}{2} \Rightarrow i$ gets b
j wins with prob. $\frac{1}{2} \Rightarrow i$ gets $-|\hat{x}_i - \hat{x}_j|$
j runs $\Rightarrow$ cost c

...ses *Out* and j wins
...ses *Run* and j wins
$b - c$ ... if i chooses *Run* and i wins
$\frac{1}{2}b - \frac{1}{2}|\hat{x}_i - \hat{x}_j| - c$ if i chooses *Run* and i and j tie for first place

Citizen-candidates

Strategic game

- Players: citizens
- For each citizen i,
  - possible actions: $\{Run, Out\}$
  - payoff:

$$\begin{cases} -|\hat{x}_i - \hat{x}_j| & \text{if } i \text{ chooses } Out \text{ and } j \text{ wins} \\ -|\hat{x}_i - \hat{x}_j| - c & \text{if } i \text{ chooses } Run \text{ and } j \text{ wins} \\ b - c & \text{if } i \text{ chooses } Run \text{ and } i \text{ wins} \\ \frac{1}{2}b - \frac{1}{2}|\hat{x}_i - \hat{x}_j| - c & \text{if } i \text{ chooses } Run \text{ and } i \text{ and } j \text{ tie for first place} \end{cases}$$

If no one enters, everyone's payoff is $K < b - c$.

Citizen-candidates

Strategic game

- Players: citizens
- For each citizen i,
  - possible actions: $\{Run, Out\}$
  - payoff:

$$\begin{cases} -|\hat{x}_i - \hat{x}_j| & \text{if } i \text{ chooses } Out \text{ and } j \text{ wins} \\ -|\hat{x}_i - \hat{x}_j| - c & \text{if } i \text{ chooses } Run \text{ and } j \text{ wins} \\ b - c & \text{if } i \text{ chooses } Run \text{ and } i \text{ wins} \\ \frac{1}{2}b - \frac{1}{2}|\hat{x}_i - \hat{x}_j| - c & \text{if } i \text{ chooses } Run \text{ and } i \text{ and } j \text{ tie for first place} \end{cases}$$

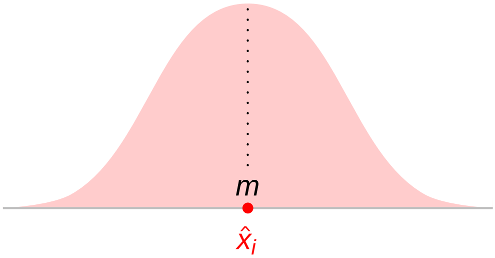
If no one enters, everyone's payoff is $K < b - c$.

Assume symmetric single-peaked distribution of favorite positions (makes some arguments easier)

Citizen-candidates

Nash equilibrium with one candidate?

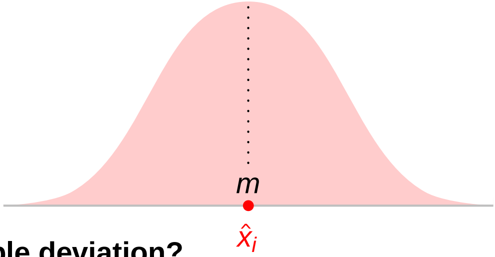
Suppose citizen i with favorite position m is only candidate



Citizen-candidates

Nash equilibrium with one candidate?

Suppose citizen i with favorite position m is only candidate



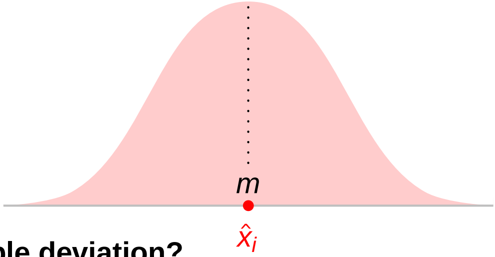
Any profitable deviation?

- i: current payoff

Citizen-candidates

Nash equilibrium with one candidate?

Suppose citizen i with favorite position m is only candidate



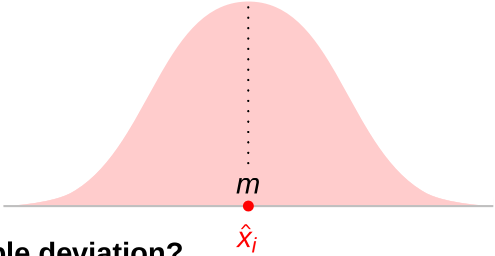
Any profitable deviation?

- i: current payoff $b - c$

Citizen-candidates

Nash equilibrium with one candidate?

Suppose citizen i with favorite position m is only candidate



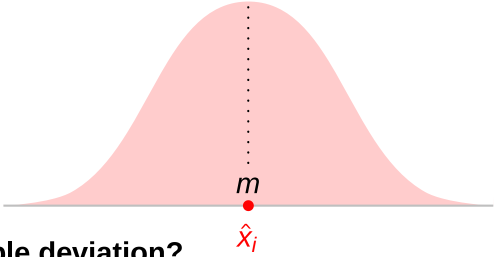
Any profitable deviation?

- i: current payoff $b - c$; withdraws $\Rightarrow$ payoff

Citizen-candidates

Nash equilibrium with one candidate?

Suppose citizen i with favorite position m is only candidate



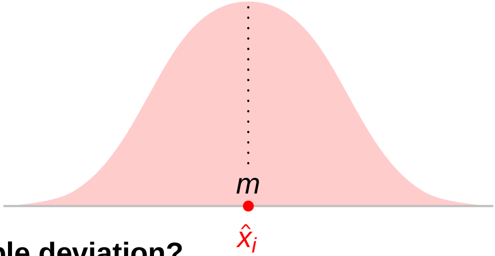
Any profitable deviation?

- i: current payoff $b - c$; withdraws $\Rightarrow$ payoff $K < b - c$

Citizen-candidates

Nash equilibrium with one candidate?

Suppose citizen i with favorite position m is only candidate



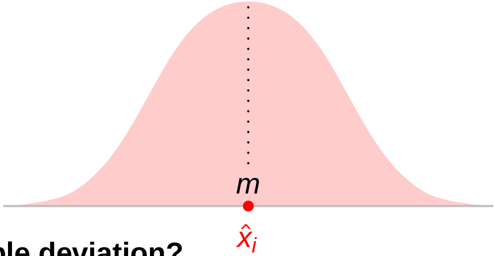
Any profitable deviation?

- i: current payoff $b - c$; withdraws $\Rightarrow$ payoff $K < b - c$
- another citizen with favorite position m: current payoff

Citizen-candidates

Nash equilibrium with one candidate?

Suppose citizen i with favorite position m is only candidate



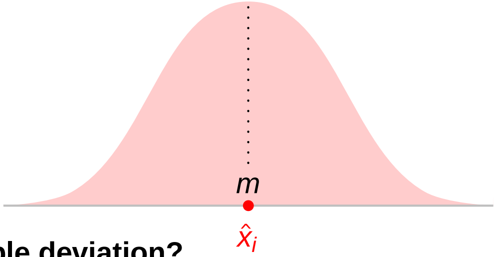
Any profitable deviation?

- i: current payoff $b - c$; withdraws $\Rightarrow$ payoff $K < b - c$
- another citizen with favorite position m: current payoff 0

Citizen-candidates

Nash equilibrium with one candidate?

Suppose citizen i with favorite position m is only candidate



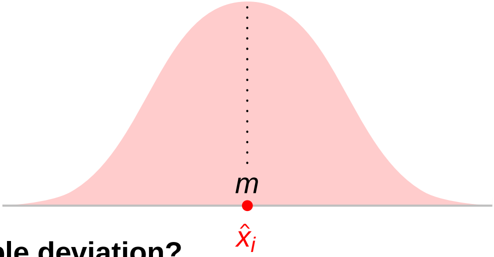
Any profitable deviation?

- i: current payoff $b - c$; withdraws $\Rightarrow$ payoff $K < b - c$
- another citizen with favorite position m: current payoff 0; enters $\Rightarrow$ payoff

Citizen-candidates

Nash equilibrium with one candidate?

Suppose citizen i with favorite position m is only candidate



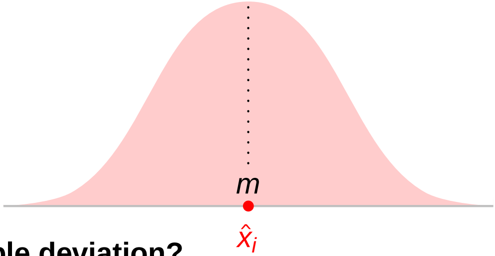
Any profitable deviation?

- i: current payoff $b - c$; withdraws $\Rightarrow$ payoff $K < b - c$
- another citizen with favorite position m: current payoff 0; enters $\Rightarrow$ payoff $\frac{1}{2}b - c$

Citizen-candidates

Nash equilibrium with one candidate?

Suppose citizen i with favorite position m is only candidate



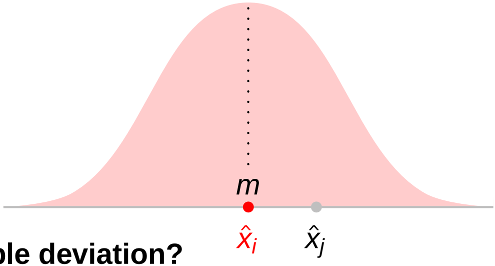
Any profitable deviation?

- i: current payoff $b - c$; withdraws $\Rightarrow$ payoff $K < b - c$
- another citizen with favorite position m: current payoff 0; enters $\Rightarrow$ payoff $\frac{1}{2}b - c \Rightarrow$ entry not profitable if $b \leq 2c$

Citizen-candidates

Nash equilibrium with one candidate?

Suppose citizen i with favorite position m is only candidate



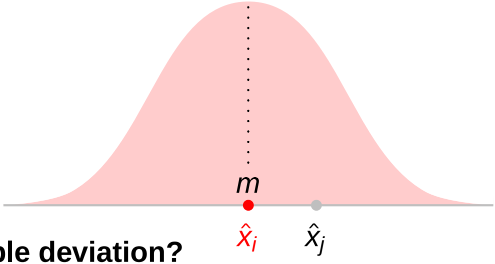
Any profitable deviation?

- i: current payoff $b - c$; withdraws $\Rightarrow$ payoff $K < b - c$
- another citizen with favorite position m: current payoff 0; enters $\Rightarrow$ payoff $\frac{1}{2}b - c \Rightarrow$ entry not profitable if $b \leq 2c$
- citizen j with favorite position $\hat{x}_j \neq m$: current payoff

Citizen-candidates

Nash equilibrium with one candidate?

Suppose citizen i with favorite position m is only candidate



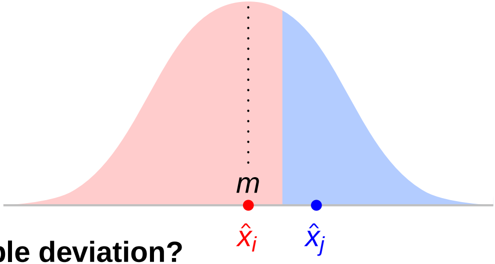
Any profitable deviation?

- i: current payoff $b - c$; withdraws $\Rightarrow$ payoff $K < b - c$
- another citizen with favorite position m: current payoff 0; enters $\Rightarrow$ payoff $\frac{1}{2}b - c \Rightarrow$ entry not profitable if $b \leq 2c$
- citizen j with favorite position $\hat{x}_j \neq m$: current payoff $-|\hat{x}_j - m|$

Citizen-candidates

Nash equilibrium with one candidate?

Suppose citizen i with favorite position m is only candidate



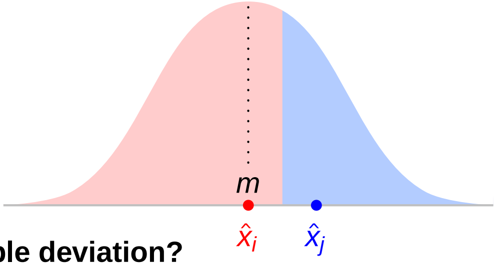
Any profitable deviation?

- i: current payoff $b - c$; withdraws $\Rightarrow$ payoff $K < b - c$
- another citizen with favorite position m: current payoff 0; enters $\Rightarrow$ payoff $\frac{1}{2}b - c \Rightarrow$ entry not profitable if $b \leq 2c$
- citizen j with favorite position $\hat{x}_j \neq m$: current payoff $-|\hat{x}_j - m|$; enters $\Rightarrow$

Citizen-candidates

Nash equilibrium with one candidate?

Suppose citizen i with favorite position m is only candidate



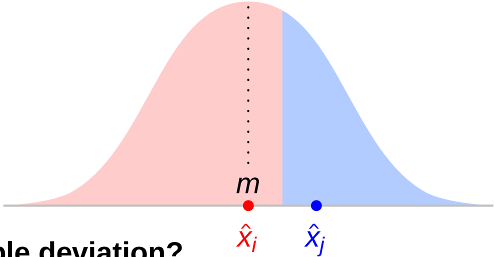
Any profitable deviation?

- i: current payoff $b - c$; withdraws $\Rightarrow$ payoff $K < b - c$
- another citizen with favorite position m: current payoff 0; enters $\Rightarrow$ payoff $\frac{1}{2}b - c \Rightarrow$ entry not profitable if $b \leq 2c$
- citizen j with favorite position $\hat{x}_j \neq m$: current payoff $-|\hat{x}_j - m|$; enters $\Rightarrow$ loses $\Rightarrow$

Citizen-candidates

Nash equilibrium with one candidate?

Suppose citizen i with favorite position m is only candidate



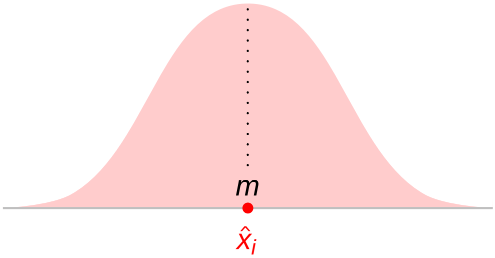
Any profitable deviation?

- i: current payoff $b - c$; withdraws $\Rightarrow$ payoff $K < b - c$
- another citizen with favorite position m: current payoff 0; enters $\Rightarrow$ payoff $\frac{1}{2}b - c \Rightarrow$ entry not profitable if $b \leq 2c$
- citizen j with favorite position $\hat{x}_j \neq m$: current payoff $-|\hat{x}_j - m|$; enters $\Rightarrow$ loses $\Rightarrow$ payoff $-|\hat{x}_j - m| - c$

Citizen-candidates

Nash equilibrium with one candidate?

Suppose citizen i with favorite position m is only candidate



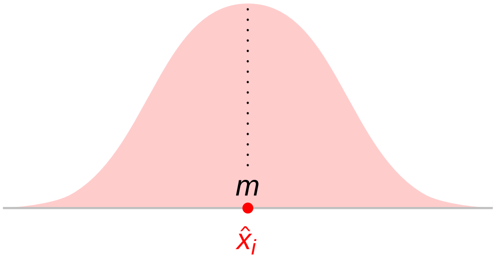
Conclusion

If $b \leq 2c$ the game has a Nash equilibrium with a single candidate whose position is m

Citizen-candidates

Nash equilibrium with one candidate?

Suppose citizen i with favorite position m is only candidate



Conclusion

If $b \leq 2c$ the game has a Nash equilibrium with a single candidate whose position is m

Under some conditions the game also has an equilibrium with a single candidate whose position is different from m (Exercise)

Citizen-candidates

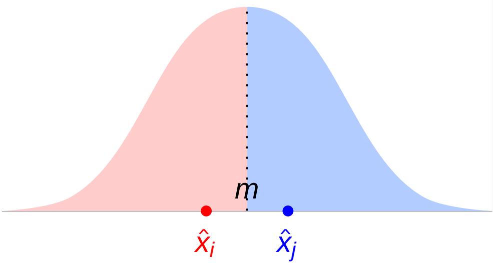
Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie $\Rightarrow$ symmetric about m

Citizen-candidates

Nash equilibrium with two candidates at different positions?

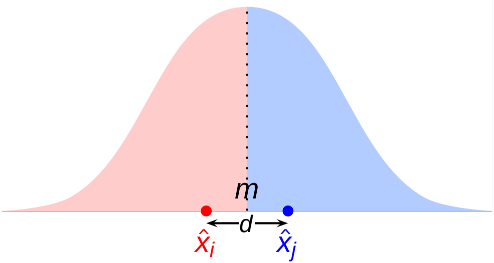
Look for NE in which candidates tie ⇒ symmetric about m



Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie $\Rightarrow$ symmetric about m

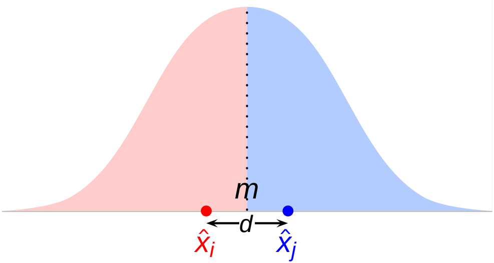


Let $d = \hat{x}_j - \hat{x}_i$ (distance between candidates' positions)

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie $\Rightarrow$ symmetric about m

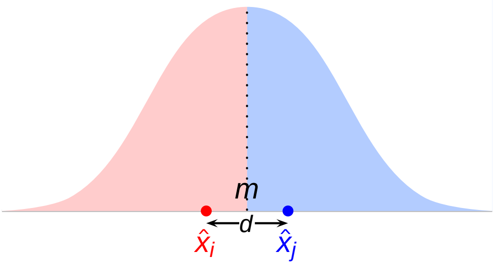


- Outcome is $\hat{x}_i$ with probability $\frac{1}{2}$ and $\hat{x}_j$ with probability $\frac{1}{2}$

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie ⇒ symmetric about m

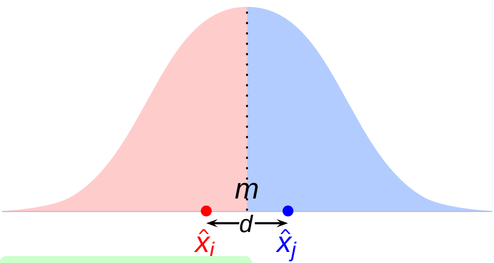


- Outcome is $\hat{x}_i$ with probability $\frac{1}{2}$ and $\hat{x}_j$ with probability $\frac{1}{2}$
- Payoff of i:

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie $\Rightarrow$ symmetric about m



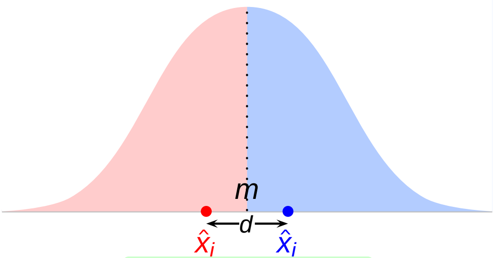
- Outcome … ty $\frac{1}{2}$ and $\hat{x}_j$ with probability $\frac{1}{2}$
- Payoff of i: $-\frac{1}{2}|\hat{x}_j - \hat{x}_i|$

j wins with prob. $\frac{1}{2}$

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie $\Rightarrow$ symmetric about m



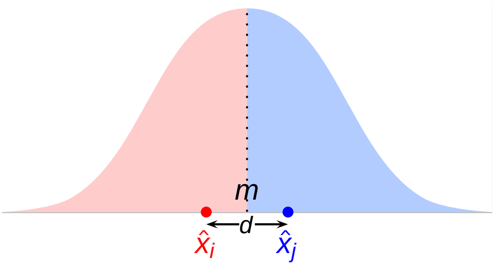
i wins with prob. $\frac{1}{2}$

- Outcome is $\hat{x}_i$ with [illegible] $\hat{x}_j$ with probability $\frac{1}{2}$
- Payoff of i: $-\frac{1}{2}|\hat{x}_j - \hat{x}_i| + \frac{1}{2}b$

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie $\Rightarrow$ symmetric about m



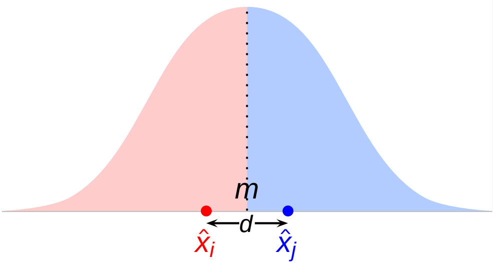
- Outcome is $\hat{x}_i$ with probabi[entry cost] $_j$ with probability $\frac{1}{2}$
- Payoff of i: $-\frac{1}{2}|\hat{x}_j - \hat{x}_i| + \frac{1}{2}b - c$

entry cost

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie $\Rightarrow$ symmetric about m

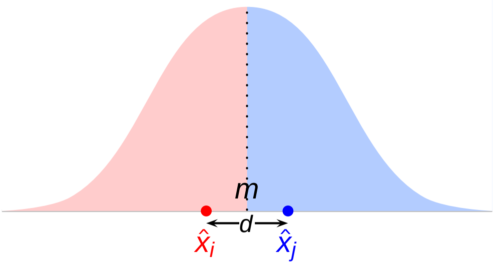


- Outcome is $\hat{x}_i$ with probability $\frac{1}{2}$ and $\hat{x}_j$ with probability $\frac{1}{2}$
- Payoff of i: $-\frac{1}{2}|\hat{x}_j - \hat{x}_i| + \frac{1}{2}b - c = -\frac{1}{2}d + \frac{1}{2}b - c$

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie $\Rightarrow$ symmetric about m

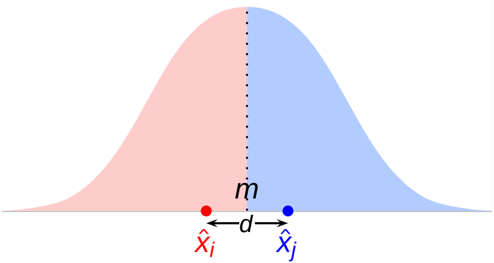


- Outcome is $\hat{x}_i$ with probability $\frac{1}{2}$ and $\hat{x}_j$ with probability $\frac{1}{2}$
- Payoff of i: $-\frac{1}{2}|\hat{x}_j - \hat{x}_i| + \frac{1}{2}b - c = -\frac{1}{2}d + \frac{1}{2}b - c$
- Payoff of j:

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie $\Rightarrow$ symmetric about m

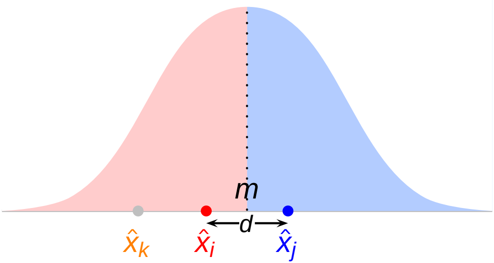


- Outcome is $\hat{x}_i$ with probability $\frac{1}{2}$ and $\hat{x}_j$ with probability $\frac{1}{2}$
- Payoff of i: $-\frac{1}{2}|\hat{x}_j - \hat{x}_i| + \frac{1}{2}b - c = -\frac{1}{2}d + \frac{1}{2}b - c$
- Payoff of j: $-\frac{1}{2}|\hat{x}_i - \hat{x}_j| + \frac{1}{2}b - c = -\frac{1}{2}d + \frac{1}{2}b - c$

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie $\Rightarrow$ symmetric about m

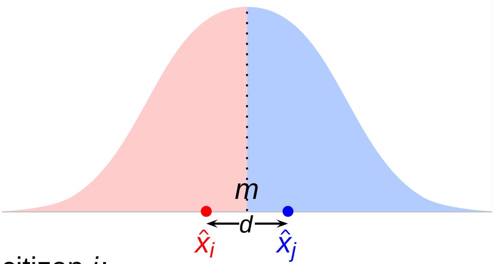


- Outcome is $\hat{x}_i$ with probability $\frac{1}{2}$ and $\hat{x}_j$ with probability $\frac{1}{2}$
- Payoff of i: $-\frac{1}{2}|\hat{x}_j - \hat{x}_i| + \frac{1}{2}b - c = -\frac{1}{2}d + \frac{1}{2}b - c$
- Payoff of j: $-\frac{1}{2}|\hat{x}_i - \hat{x}_j| + \frac{1}{2}b - c = -\frac{1}{2}d + \frac{1}{2}b - c$
- Payoff of any other citizen k: $-\frac{1}{2}|\hat{x}_k - \hat{x}_i| - \frac{1}{2}|\hat{x}_k - \hat{x}_j|$

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie $\Rightarrow$ symmetric about m



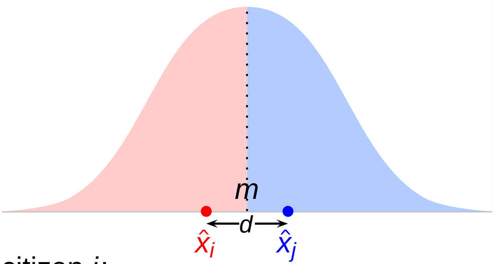
Deviation by citizen i:

- Current payoff: $-\frac{1}{2}d + \frac{1}{2}b - c$

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie $\Rightarrow$ symmetric about m



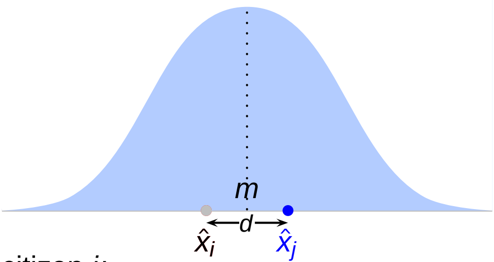
Deviation by citizen i:

- Current payoff: $-\frac{1}{2}d + \frac{1}{2}b - c$
- Exit $\Rightarrow$

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie $\Rightarrow$ symmetric about m



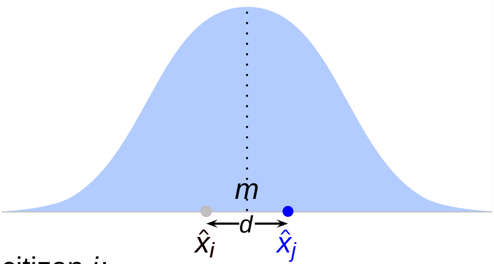
Deviation by citizen i:

- Current payoff: $-\frac{1}{2}d + \frac{1}{2}b - c$
- Exit $\Rightarrow$ outcome $\hat{x}_j$ $\Rightarrow$ payoff

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie $\Rightarrow$ symmetric about m



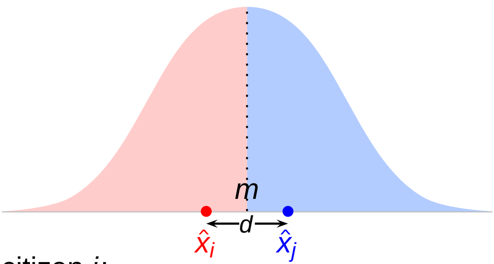
Deviation by citizen i:

- Current payoff: $-\frac{1}{2}d + \frac{1}{2}b - c$
- Exit $\Rightarrow$ outcome $\hat{x}_j \Rightarrow$ payoff $-|\hat{x}_j - \hat{x}_i| = -d$

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie $\Rightarrow$ symmetric about m



Deviation by citizen i:

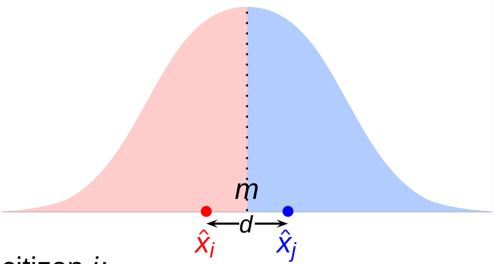
- Current payoff: $-\frac{1}{2}d + \frac{1}{2}b - c$
- Exit $\Rightarrow$ outcome $\hat{x}_j \Rightarrow$ payoff $-|\hat{x}_j - \hat{x}_i| = -d$
- So for entry to be optimal,

$$-\frac{1}{2}d + \frac{1}{2}b - c \geq -d$$

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie $\Rightarrow$ symmetric about m



Deviation by citizen i:

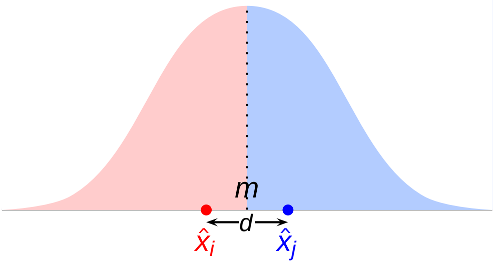
- Current payoff: $-\frac{1}{2}d + \frac{1}{2}b - c$
- Exit $\Rightarrow$ outcome $\hat{x}_j$ $\Rightarrow$ payoff $-|\hat{x}_j - \hat{x}_i| = -d$
- So for entry to be optimal,

$$-\tfrac{1}{2}d + \tfrac{1}{2}b - c \geq -d$$
$$\Rightarrow d \geq 2c - b$$

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie ⇒ symmetric about m

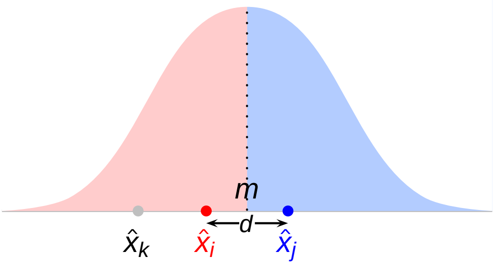


Deviation by citizen j: Same argument as for citizen i

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie $\Rightarrow$ symmetric about m

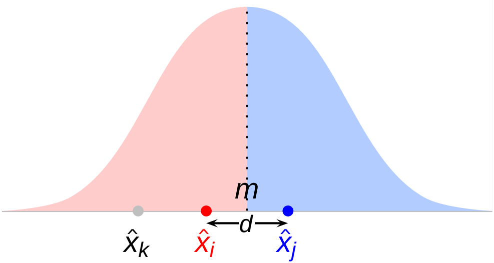


Deviation by citizen k with favorite position $\hat{x}_k \leq \hat{x}_i$:

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie $\Rightarrow$ symmetric about m



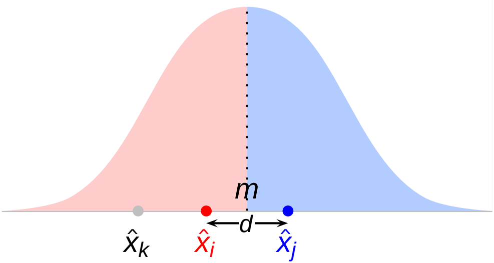
Deviation by citizen k with favorite position $\hat{x}_k \leq \hat{x}_i$:

- Current payoff: $-\frac{1}{2}|\hat{x}_k - \hat{x}_i| - \frac{1}{2}|\hat{x}_k - \hat{x}_j|$

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie ⇒ symmetric about m



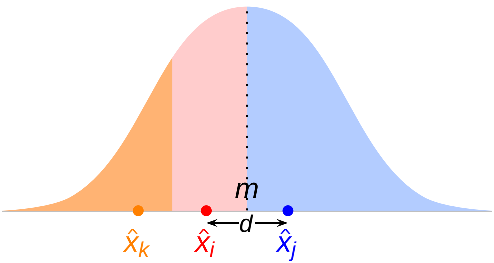
Deviation by citizen k with favorite position $\hat{x}_k \leq \hat{x}_i$:

- Current payoff: $-\frac{1}{2}|\hat{x}_k - \hat{x}_i| - \frac{1}{2}|\hat{x}_k - \hat{x}_j|$
- Enter ⇒

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie $\Rightarrow$ symmetric about m



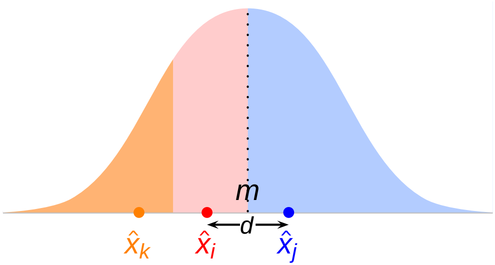
Deviation by citizen k with favorite position $\hat{x}_k \leq \hat{x}_i$:

- Current payoff: $-\frac{1}{2}|\hat{x}_k - \hat{x}_i| - \frac{1}{2}|\hat{x}_k - \hat{x}_j|$
- Enter $\Rightarrow$ winner is j

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie ⇒ symmetric about m



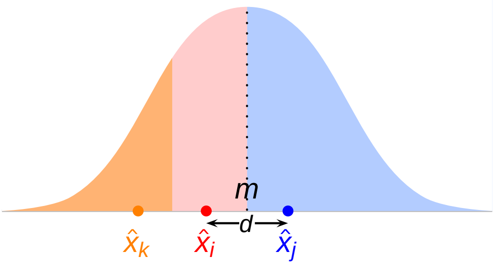
Deviation by citizen k with favorite position $\hat{x}_k \leq \hat{x}_i$:

- Current payoff: $-\frac{1}{2}|\hat{x}_k - \hat{x}_i| - \frac{1}{2}|\hat{x}_k - \hat{x}_j|$
- Enter ⇒ winner is j
  ⇒ payoff

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie $\Rightarrow$ symmetric about m



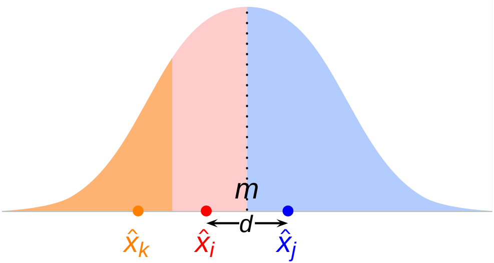
Deviation by citizen k with favorite position $\hat{x}_k \leq \hat{x}_i$:

- Current payoff: $-\frac{1}{2}|\hat{x}_k - \hat{x}_i| - \frac{1}{2}|\hat{x}_k - \hat{x}_j|$
- Enter $\Rightarrow$ winner is j
  $\Rightarrow$ payoff $-|\hat{x}_j - \hat{x}_k| - c$

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie $\Rightarrow$ symmetric about m



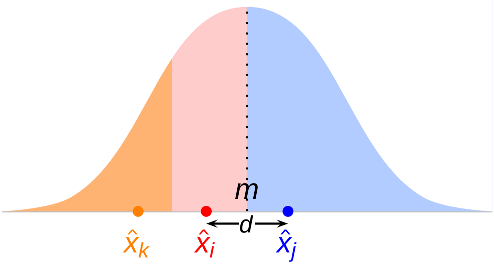
Deviation by citizen k with favorite position $\hat{x}_k \le \hat{x}_i$:

- Current payoff: $-\frac{1}{2}|\hat{x}_k - \hat{x}_i| - \frac{1}{2}|\hat{x}_k - \hat{x}_j|$
- Enter $\Rightarrow$ winner is j
  $\Rightarrow$ payoff $-|\hat{x}_j - \hat{x}_k| - c < -\frac{1}{2}|\hat{x}_k - \hat{x}_i| - \frac{1}{2}|\hat{x}_k - \hat{x}_j|$

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie $\Rightarrow$ symmetric about m



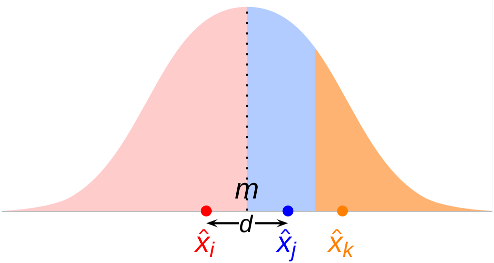
Deviation by citizen k with favorite position $\hat{x}_k \leq \hat{x}_i$:

- Current payoff: $-\frac{1}{2}|\hat{x}_k - \hat{x}_i| - \frac{1}{2}|\hat{x}_k - \hat{x}_j|$
- Enter $\Rightarrow$ winner is j
  $\Rightarrow$ payoff $-|\hat{x}_j - \hat{x}_k| - c < -\frac{1}{2}|\hat{x}_k - \hat{x}_i| - \frac{1}{2}|\hat{x}_k - \hat{x}_j|$
  $\Rightarrow$ k is worse off

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie ⇒ symmetric about m

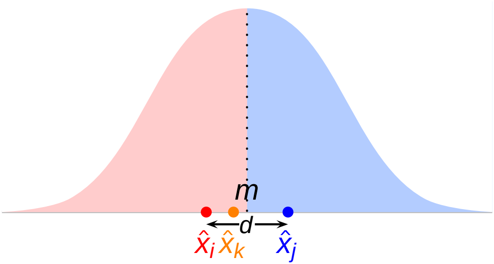


Entry by citizen k with favorite position $\hat{x}_k \geq \hat{x}_j$: same argument

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie ⇒ symmetric about m

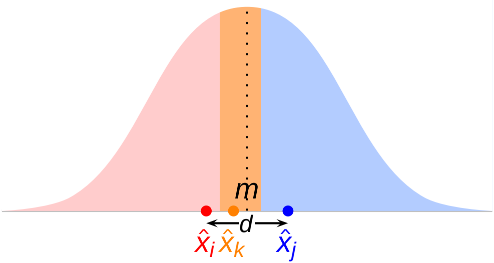


Entry by citizen k with favorite position $\hat{x}_k$, where $\hat{x}_i < \hat{x}_k < m$:

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie $\Rightarrow$ symmetric about m



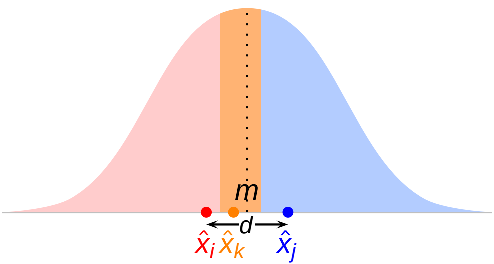
Entry by citizen k with favorite position $\hat{x}_k$, where $\hat{x}_i < \hat{x}_k < m$:

- If $\hat{x}_i$ and $\hat{x}_j$ are close enough, j wins

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie $\Rightarrow$ symmetric about m



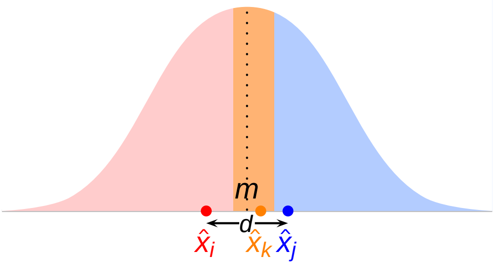
Entry by citizen k with favorite position $\hat{x}_k$, where $\hat{x}_i < \hat{x}_k < m$:

- If $\hat{x}_i$ and $\hat{x}_j$ are close enough, j wins
  $\Rightarrow$ k is worse off (because winning position is worse and pays entry cost c)

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie ⇒ symmetric about m

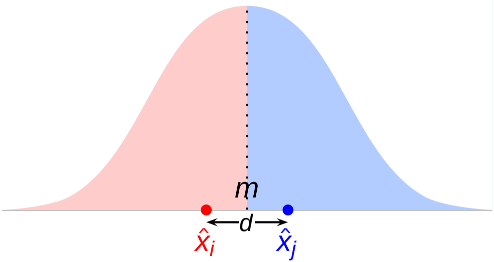


Deviation by citizen k with favorite position $\hat{x}_k$, where $m < \hat{x}_k < \hat{x}_j$: same argument

Citizen-candidates

Nash equilibrium with two candidates at different positions?

Look for NE in which candidates tie ⇒ symmetric about m



Conclusion If distance between candidates is at least $2c - b$ but not big enough that a citizen who enters between them can win, the configuration is an equilibrium

Citizen-candidates

Nash equilibria with one and two candidates: summary

For a symmetric single-peaked distribution of favorite positions,

Citizen-candidates

Nash equilibria with one and two candidates: summary

For a symmetric single-peaked distribution of favorite positions,

- if $b \leq 2c$ then there is an equilibrium with a single candidate

Citizen-candidates

Nash equilibria with one and two candidates: summary

For a symmetric single-peaked distribution of favorite positions,

- if $b \leq 2c$ then there is an equilibrium with a single candidate
- there are equilibria with two candidates symmetrically located around the median favorite position, not too close together and not too far apart

Electoral competition: summary

Parties that care only about winning

Two parties Unique Nash equilibrium, in which position of both parties is median of citizens' favorite positions

Electoral competition: summary

Parties that care only about winning

Two parties Unique Nash equilibrium, in which position of both parties is median of citizens' favorite positions

Three parties No Nash equilibrium

Electoral competition: summary

Parties that care only about winning

Two parties Unique Nash equilibrium, in which position of both parties is median of citizens' favorite positions

Three parties No Nash equilibrium

Parties that care only about winning policy

Electoral competition: summary

Parties that care only about winning

Two parties Unique Nash equilibrium, in which position of both parties is median of citizens' favorite positions

Three parties No Nash equilibrium

Parties that care only about winning policy

Two parties Unique Nash equilibrium, in which position of both parties is median of citizens' favorite positions

Electoral competition: summary

Parties that care only about winning

Two parties Unique Nash equilibrium, in which position of both parties is median of citizens' favorite positions

Three parties No Nash equilibrium

Parties that care only about winning policy

Two parties Unique Nash equilibrium, in which position of both parties is median of citizens' favorite positions

Citizen-candidates

Nash equilibria with one, two, and more candidates. Equilibrium positions may be dispersed.