Economics 316

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Solutions to problems for Tutorial 2

1. (a) Yes, (c, c) is still a Nash equilibrium, by the same argument as before.

   (b) In addition, $(c+1, c+1)$ is a Nash equilibrium (where c is given in cents). In this equilibrium both firms' profits are positive. If either firm raises its price or lowers it to c, its profit becomes zero. If either firm lowers its price below c, its profit becomes negative.

   No other pair of prices is a Nash equilibrium, by the following argument, similar to the argument I gave in class (which is in the text) for the case in which a price can be any nonnegative number.

   - If $p_i < c$ then the firm whose price is lowest (or either firm, if the prices are the same) can increase its profit (to zero) by raising its price to c.
   - If $p_i = c$ and $p_j \geq c+1$ then firm i can increase its profit from zero to a positive amount by increasing its price to $c+1$.
   - If $p_i > p_j \geq c+1$ then firm i can increase its profit (from zero) by lowering its price to $c+1$.
   - If $p_i = p_j \geq c+2$ and $p_j < \alpha$ then either firm can increase its profit by lowering its price by one cent. (If firm i does so, its profit changes from $\frac{1}{2}(p_i - c)(\alpha - p_i)$ to $(p_i - 1 - c)(\alpha - p_i + 1) = (p_i - 1 - c)(\alpha - p_i) + p_i - 1 - c$. We have $p_i - 1 - c \geq \frac{1}{2}(p_i - c)$ and $p_i - 1 - c > 0$, since $p_i \geq c+2$.)
   - If $p_i = p_j \geq c+2$ and $p_j \geq \alpha$ then either firm can increase its profit by lowering its price to p^m.

2. Consider a profile $(p_1, \ldots, p_n)$ of prices in which $p_i \geq c$ for all i and at least two prices are equal to c. Every firm's profit is zero. If any firm raises its price its profit remains zero. If a firm charging more than c lowers its price, but not below c, its profit also remains zero. If a firm lowers its price below c then its profit is negative. Thus any such profile is a Nash equilibrium.

   To show that no other profile is a Nash equilibrium, we can argue as follows.

- If some price is less than c then the firm charging the lowest price can increase its profit (to zero) by increasing its price to c.
- If exactly one firm's price is equal to c then that firm can increase its profit by raising its price a little (keeping it less than the next highest price).
- If all firms' prices exceed c then the firm charging the highest price can increase its profit by lowering its price to some price between c and the lowest price being charged.

3. Firm 1's profit is

$$\pi_1(q_1, q_2) = \begin{cases} q_1(\alpha - q_1 - q_2) - q_1^2 & \text{if } q_1 + q_2 \leq \alpha \\ -q_1^2 & \text{if } q_1 + q_2 > \alpha \end{cases}$$

or

$$\pi_1(q_1, q_2) = \begin{cases} q_1(\alpha - 2q_1 - q_2) & \text{if } q_1 + q_2 \leq \alpha \\ -q_1^2 & \text{if } q_1 + q_2 > \alpha. \end{cases}$$

When it is positive, this function is a quadratic in q_1 that is zero at $q_1 = 0$ and at $q_1 = (\alpha - q_2)/2$. Thus firm 1's best response function is

$$b_1(q_2) = \begin{cases} \frac{1}{4}(\alpha - q_2) & \text{if } q_2 \leq \alpha \\ 0 & \text{if } q_2 > \alpha. \end{cases}$$

Since the firms' cost functions are the same, firm 2's best response function is the same as firm 1's: $b_2(q) = b_1(q)$ for all q. The firms' best response functions are shown in Figure 1.

Solving the two equations $q_1^* = b_1(q_2^*)$ and $q_2^* = b_2(q_1^*)$ we find that there is a unique Nash equilibrium, in which the output of firm i ($i = 1, 2$) is $q_i^* = \frac{1}{5}\alpha$.

4. - If $P(Q^*) < \underline{p}$ then every firm producing a positive output makes a negative profit, and can increase its profit (to 0) by deviating and producing zero.
   - If $P(Q^* + \underline{q}) > \underline{p}$, take a firm that is either producing no output, or an arbitrarily small output. (For any $\varepsilon > 0$ a firm producing less than ε exists because demand is finite.) Such a firm earns a profit of either zero or arbitrarily close to zero. If it deviates and chooses the output $\underline{q}$ then total output changes to at most $Q^* + \underline{q}$, so that the price still exceeds $\underline{p}$ (because $P(Q^* + \underline{q}) > \underline{p}$). Hence the deviant makes a positive profit.

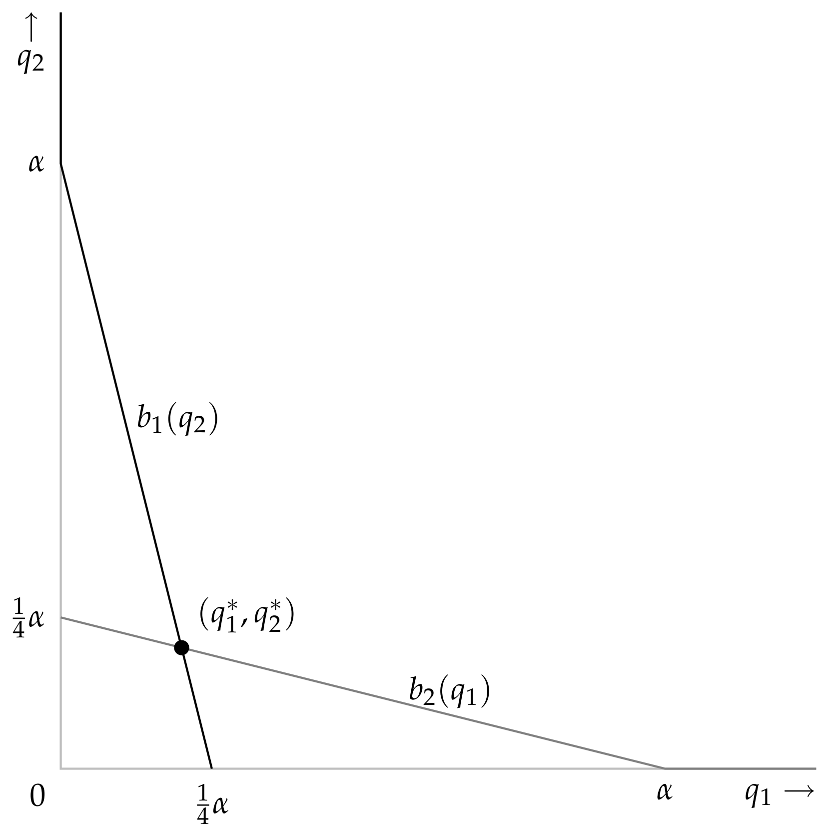

Figure 1. The best response functions in Cournot's duopoly game with linear inverse demand and a quadratic cost function. The unique Nash equilibrium is $(q_1^*, q_2^*) = (\frac{1}{5}\alpha, \frac{1}{5}\alpha)$.