Economics 316

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Problems for Tutorial 2

1. Consider a variant of the example of Bertrand's duopoly game considered in class in which each firm is restricted to choose a price that is an integral number of cents. Take the monetary unit to be a cent, and assume that c is an integer and $\alpha > c + 1$.

   (a) Is (c, c) a Nash equilibrium of this game?

   (b) Does the game have any other Nash equilibrium?

2. Consider a generalization of the example of Bertrand's model considered in class in which there are n firms, with $n \geq 3$. (Maintain the assumptions made in class on the cost and demand functions.) Find the set of Nash equilibria of the resulting strategic game.

3. Find the Nash equilibrium of Cournot's game when there are two firms, the inverse demand function is the same as the one in the example considered class, and the cost function of each firm i is $C_i(q_i) = q_i^2$.

4. Consider a variant of the n-firm version of Cournot's model in which each firm's average cost function is U-shaped, rather than being constant. Suppose that there are infinitely many firms, all of which have the same cost function C. Assume that $C(0) = 0$, and that for $q > 0$ the function $C(q)/q$ (the average cost function) has a unique minimizer $\underline{q}$; denote the minimum of $C(q)/q$ by $\underline{p}$. (See Figure 1.) Assume that the inverse demand function P is decreasing.

   Show that in any Nash equilibrium of the game the firms' total output Q^* satisfies

   $$P(Q^* + \underline{q}) \leq \underline{p} \leq P(Q^*).$$

   (That is, the price is at least the minimal value $\underline{p}$ of the average cost, but is close enough to this minimum that increasing the total output of the firms by $\underline{q}$ would reduce the price to at most $\underline{p}$.)

   To establish these inequalities, show that if $P(Q^*) < \underline{p}$ or $P(Q^* + \underline{q}) > \underline{p}$, then Q^* is not the total output of the firms in a Nash equilibrium, because in each case at least one firm can deviate and increase its profit.

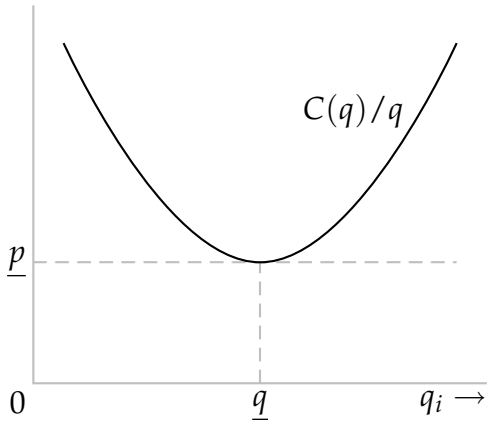

Figure 1. The average cost function of each firm in Problem 4.

(You need to identify the firm that can profitably deviate and the deviation it can profitably make.)