Economics 316

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Problem Set 2

1. Consider a variant of the investment game considered in class in which each person chooses how much effort to expend (rather than simply choosing whether to invest) and $k = n$ (i.e. everyone's participation is required for the project to succeed). The amount of effort of each person is restricted to be a positive integer at most equal to K. Denote person i's effort by e_i. The outcome of the project's depends on the *smallest* of all the efforts, $\min_j e_j$. ("A chain is as strong as its weakest link.") Specifically, person i's payoff to the action profile $(e_1, \ldots, e_n)$ is $2\min_j e_j - e_i$.

   (a) Is there any value of e for which the action profile $(e, \ldots, e)$, in which every person devotes the same effort to the project, is a Nash equilibrium? (What is a player's payoff to this profile? What is her payoff if she deviates to a lower or higher effort level?)

   (b) Is any action profile in which not all the players' effort levels are the same a Nash equilibrium? (Consider a player whose effort exceeds the minimum effort level of all players. What happens to her payoff if she reduces her effort level to the minimum?)

2. Consider a generalization of the example of Bertrand's model considered in class in which the demand function D is not necessarily linear. Suppose that D is any continuous function for which $D(p) \geq 0$ for all p and there exists $\overline{p} > c$ such that $D(p) > 0$ for all $p \leq \overline{p}$. (A function is "continuous" if it has no jumps.) Is (c, c) still a Nash equilibrium? Is it still the only Nash equilibrium?

3. Consider a variant of the example of Bertrand's duopoly game considered in class in which, as in the first Tutorial problem, each firm is restricted to choose a price that is an integral number of cents, *and* the firms' unit costs differ. Take the monetary unit to be a cent, and assume that the unit cost of firm 1 is 100 (i.e. \$1) and the unit cost of firm 2 is 200. Assume also that $\alpha > 300$. Find the set of Nash equilibria of the resulting strategic game.

4. Consider a variant of the example of Cournot's duopoly game considered in class in which the firms' unit costs differ. Denote the unit cost of firm i by c_i and assume $c_1 > c_2$, where $c_1 < \alpha$.

    (a) Find the Nash equilibrium of this game. (There are two cases, depending on the size of c_1 relative to c_2.)

    (b) Which firm produces more output in the equilibrium?

    (c) What is the effect of technical change that lowers firm 2's unit cost c_2 (while not affecting firm 1's unit cost c_1) on the firms' equilibrium outputs, the total output, and the price?

5. Two players can contribute to a public good. If the pair of contributions is (c_1, c_2) then player i's payoff, for $i = 1, 2$, is $C - C^2 - (c_i)^2$, where $C = c_1 + c_2$. Each player's contribution can be any nonnegative number. Find the Nash equilibrium (equilibria?) of the strategic game that models this situation.

6. Two firms produce goods that are partially (not perfectly) substitutable. When the prices set by the firms are p_1 and p_2, the demand faced by firm 1 is $10 - p_1 + 2p_2$ and the demand faced by firm 2 is $20 - p_2 + \frac{1}{2}p_1$. Each firm's cost of production is zero. Model the competition between the firms as a strategic game in which each firm chooses a price and find the Nash equilibrium (equilibria?) of the game.