ECO316: Applied game theory

Lecture 2

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Example: Investing in a joint project

- n people

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- Noninvestors unaffected by outcome of project
- So for every person,

  invest & project succeeds $\succ$ don't invest $\succ$ invest & project fails

Investing in a joint project

Strategic game

- Players:

Investing in a joint project

Strategic game

- Players: n people

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if player chooses *Don't invest*, 0 regardless of others' actions

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Nash equilibrium

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- some other number of people invest?

Investing in a joint project

Reminder of payoffs:

- *Invest* $\Rightarrow$ 100 if $\geq k$ investors, -10 if $< k$ investors
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Summary

Exactly two Nash equilibria:

- everyone invests
- no one invests

Traveler's Dilemma

- Airline has lost suitcases of two travelers

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$$\left\{ \right.$$

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where a_i is i's action and a_j is the other player's action

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Traveler's Dilemma

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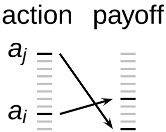
- $a_i < a_j$?



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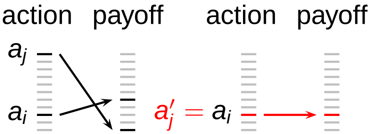

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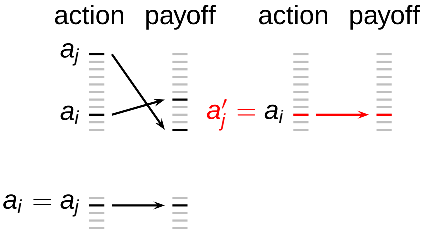

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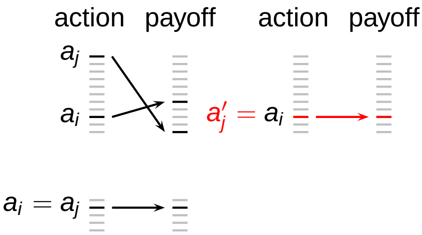

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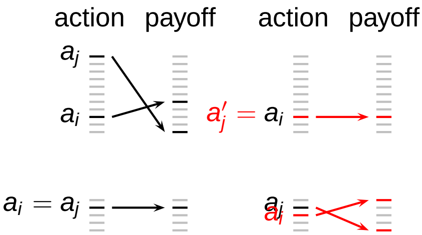

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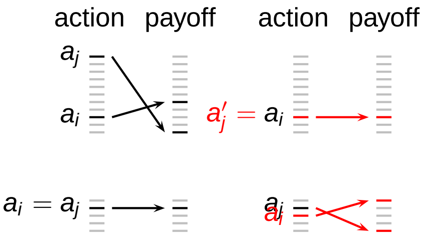

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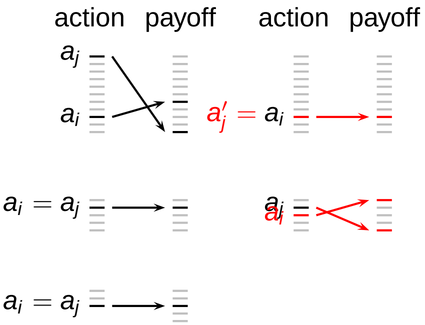
  - If $a_i = a_j = 2$, NE! If either player increases amount, payoff $= 0$



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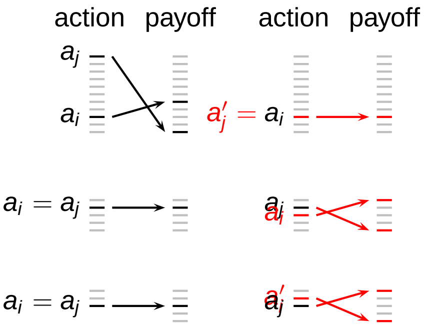

Traveler's Dilemma

Summary

Unique Nash equilibrium: both travelers name the lowest possible valuation, $2

Competition between firms

- Topic at heart of classical economic theory

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- Is price-taking assumption reasonable if number of firms is large?
- Need model in which each firm takes others into account
- Can study impact on number of firms on the outcome

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- Firms producing same good compete for customers
- Each firm's profit depends on behavior of all firms
- Model interaction between firms as strategic game
- What are properties of Nash equilibrium?
- How is Nash equilibrium related to "competitive" outcome? How does it depend on number of firms?

Bertrand's model

- Each firm chooses a unit price

Joseph Louis François Bertrand
1822–1900

Bertrand's model

- Each firm chooses a unit price
- Each firm produces enough output to satisfy demand it faces, given prices charged by all firms

Bertrand's model

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- Consumers patronize only firm with lowest price

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Strategic game

- Players:

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Strategic game

- Players: firms

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- Players: firms
- For each firm
  - possible actions:

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Strategic game

- Players: firms
- For each firm
  - possible actions: prices
  - payoff: profit

Example of Bertrand's game

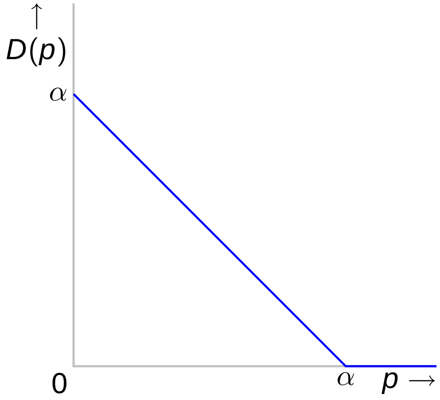
- Two firms

Example of Bertrand's game

- Two firms
- Constant unit cost, same for both firms:
  $C_i(q_i) = cq_i$ where $c > 0$ and q_i is output of firm i

Example of Bertrand's game

- Two firms
- Constant unit cost, same for both firms: $C_i(q_i) = cq_i$ where $c > 0$ and q_i is output of firm i
- Linear demand function: $D(p) = \alpha - p$ for $p \leq \alpha$



Example of Bertrand's game

- Two firms
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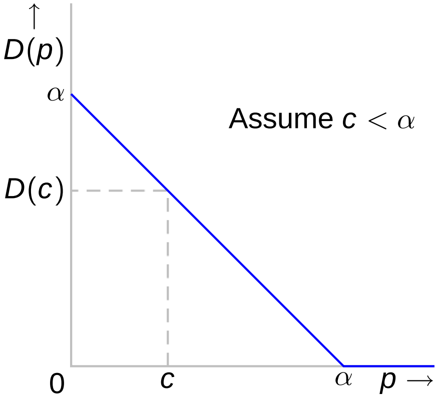

Example of Bertrand's game

Cost of firm i's producing q_i units: $C_i(q_i) = cq_i$
Total demand when price is p: $D(p) = \alpha - p$ (for $p \leq \alpha$)

Strategic game

- Players:

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Total demand when price is p: $D(p) = \alpha - p$ (for $p \leq \alpha$)

Strategic game

- Players: two firms
- For each firm i,
  - possible actions: prices (nonnegative numbers)

Price can be any number—not restricted to multiples of discrete unit (e.g. multiples of a cent)

Example of Bertrand's game

Cost of firm i's producing q_i units: $C_i(q_i) = cq_i$
Total demand when price is p: $D(p) = \alpha - p$ (for $p \leq \alpha$)

Strategic game

- Players: two firms
- For each firm i,
  - possible actions: prices (nonnegative numbers)
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where j is the other firm ($j = 2$ if $i = 1$, and $j = 1$ if $i = 2$).

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  - possibl ... ative numbers)
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Revenue from selling total demand at price p_i

$$\pi_i(p_1, p_2) = \begin{cases} p_i D(p_i) & \text{if } p_i < p_j \\ & \\ & \end{cases}$$

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Strategic game

- Players: two firms
- For each firm i,
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  - payoff: profit, whi demand at price p_i]

Cost of producing total demand at price p_i

$$\pi_i(p_1, p_2) = \begin{cases} p_i D(p_i) - cD(p_i) & \text{if } p_i < p_j \\ & \\ & \end{cases}$$

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Simplify expression

$$\pi_i(p_1, p_2) = \begin{cases} (p_i - c)D(p_i) & \text{if } p_i < p_j \\ & \\ & \end{cases}$$

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Cost of firm i's producing q_i units: $C_i(q_i) = cq_i$
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$$\pi_i(p_1, p_2) = \begin{cases} (p_i - c)(\alpha - p_i) & \text{if } p_i < p_j \\ & \\ & \end{cases}$$

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$$\pi_i(p_1, p_2) = \begin{cases} (p_i - c)(\alpha - p_i) & \text{if } p_i < p_j \\ & \\ & \text{if } p_i > p_j \end{cases} \quad \text{(assuming } p_i \leq \alpha\text{)}$$

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High price $\Rightarrow$ no customers

$\pi_i(p_1, p_2)$ … $\alpha - p_i)$ if $p_i < p_j$ … 0 if $p_i > p_j$ (assuming $p_i \leq \alpha$)

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Equal prices ⇒ demand split equally

$$\pi_i(p_1, p_2) = \begin{cases} (p_i - c)(\alpha - p_i) & \text{if } p_i < p_j \\ \frac{1}{2}(p_i - c)(\alpha - p_i) & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j \end{cases} \quad \text{(assuming } p_i \leq \alpha\text{)}$$

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Example of Bertrand's game

Exploration of payoffs:

$$\pi_i(p_1, p_2) = \begin{cases} (p_i - c)(\alpha - p_i) & \text{if } p_i < p_j \\ \frac{1}{2}(p_i - c)(\alpha - p_i) & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j \end{cases} \quad (\text{assuming } p_i \leq \alpha)$$

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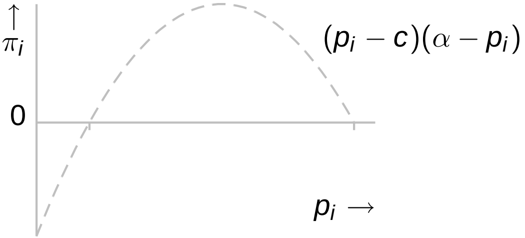

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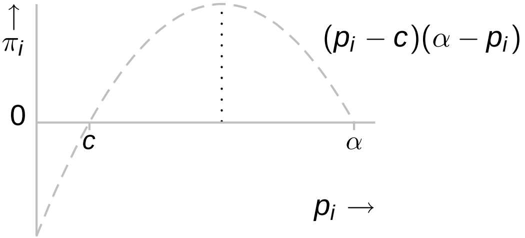

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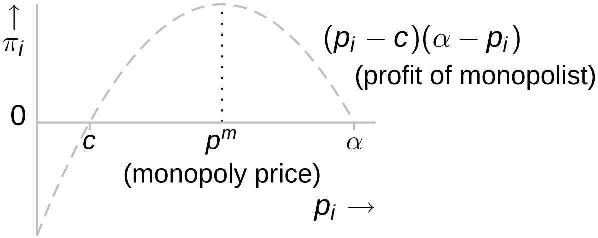

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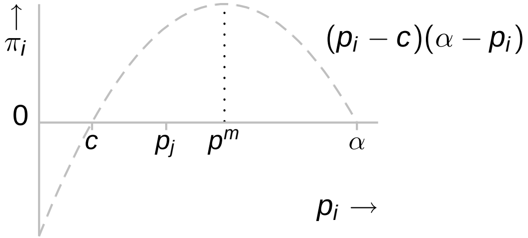

$c < p_j \leq p^m$

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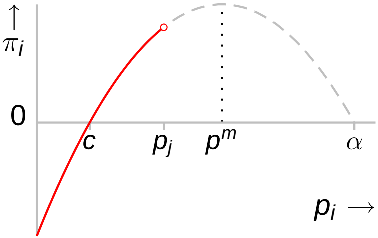

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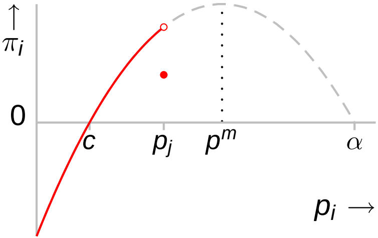

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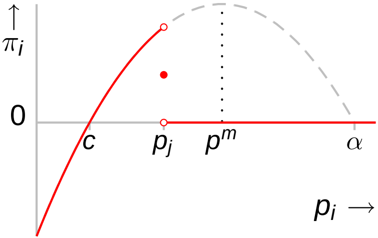

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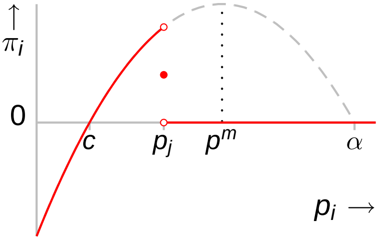

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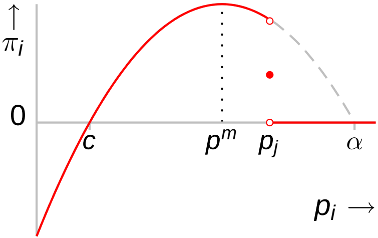
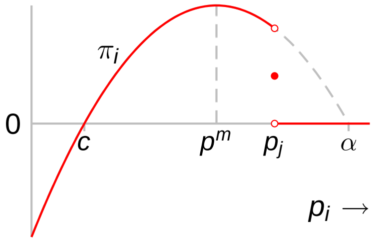


$$p^m < p_j < \alpha$$

Example of Bertrand's game

Best value of p_i given p_j?



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- If $p_j > p^m$, firm i's best price is

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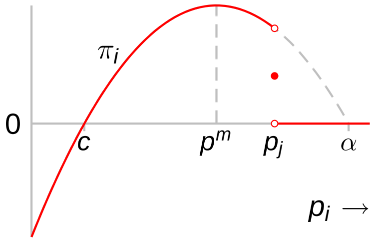

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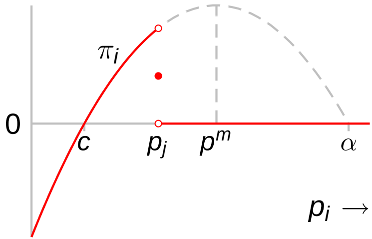

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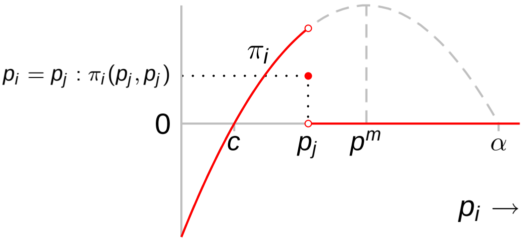

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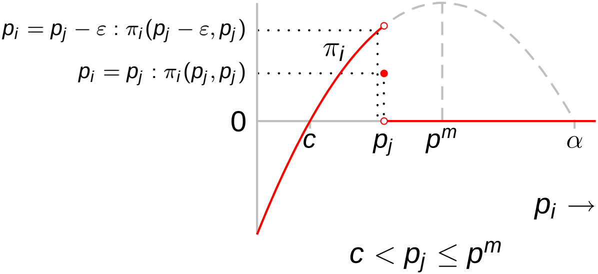

- If $p_j > p^m$, firm i's best price is p^m
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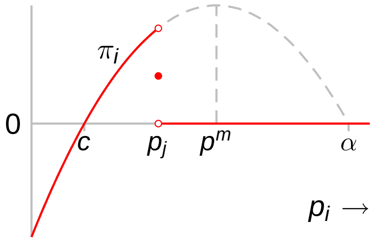

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$\Rightarrow$ incentive to "undercut" other firm's price

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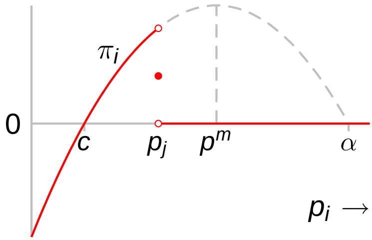

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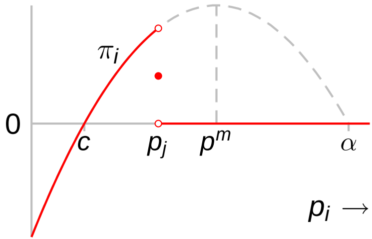

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$\Rightarrow$ incentive to "undercut" other firm's price

- Prices less than c yield losses
- So perhaps (c, c) is only equilibrium?

Example of Bertrand's game

Proof that (c, c) is a Nash equilibrium

- $\pi_1(c, c) =$

Example of Bertrand's game

Proof that (c, c) is a Nash equilibrium

- $\pi_1(c, c) = 0$

Example of Bertrand's game

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Example of Bertrand's game

Proof that (c, c) is a Nash equilibrium

- $\pi_1(c, c) = 0$
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So

$$\pi_1(c, c) \geq \pi_1(p_1, c) \text{ for all } p_1$$

and similarly for firm 2.

Example of Bertrand's game

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So

$$\pi_1(c, c) \geq \pi_1(p_1, c) \text{ for all } p_1$$

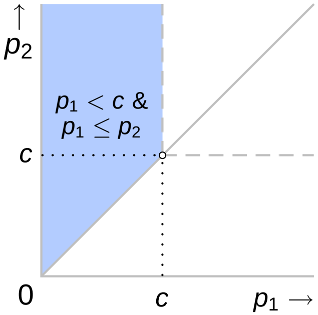
and similarly for firm 2.

Hence (c, c) is a Nash equilibrium.

Example of Bertrand's game

Proof that no pair $(p_1, p_2) \neq (c, c)$ is Nash equilibrium

- $p_1 < c$ and $p_1 \leq p_2$?



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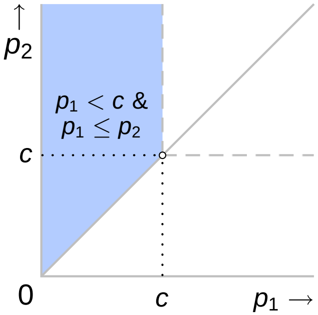

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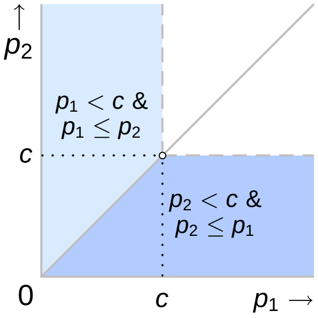
- $p_2 < c$ and $p_2 \leq p_1$?



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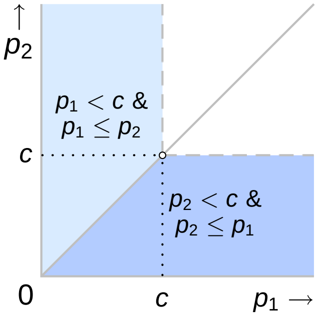

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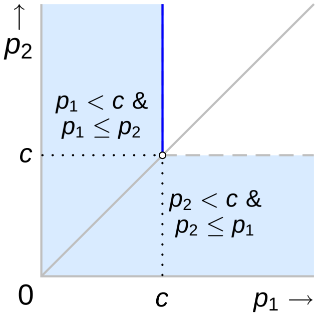

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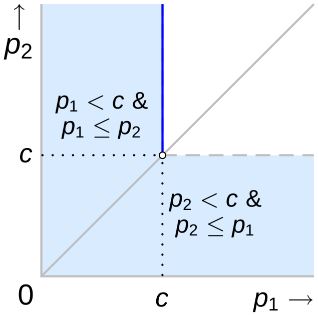

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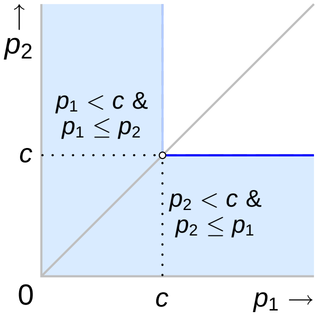

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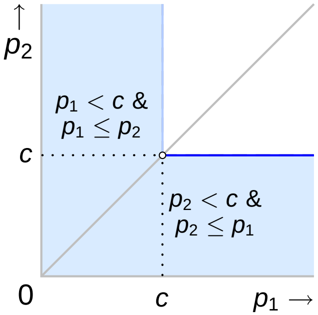

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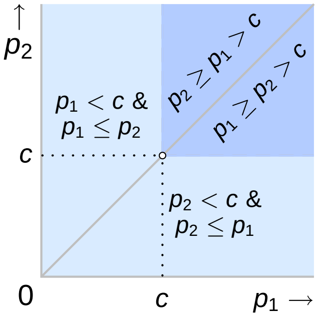

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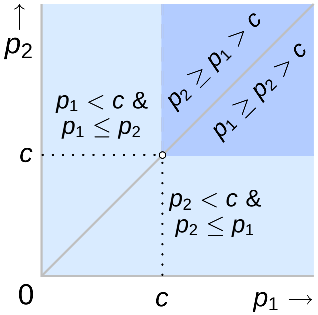

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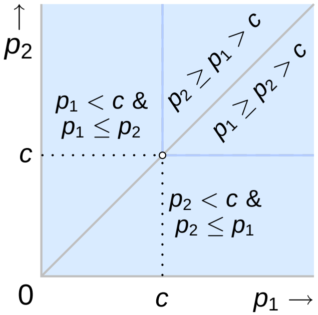

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- What if firms interact repeatedly?

Cournot's model

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Antoine Augustin Cournot
1801–1877

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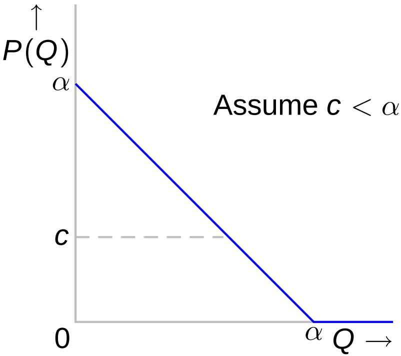

Example of Cournot's game

Cost of firm i's producing q_i units: $C_i(q_i) = cq_i$
Price when supply is Q: $P(Q) = \alpha - Q$ (for $Q \leq \alpha$)

Strategic game

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Output can be any number—not restricted to multiples of discrete unit

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$$\pi_i(q_1, q_2)$$

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$$\pi_i(q_1, q_2) = \text{revenue} - \text{cost}$$

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for $i = 1, 2$

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$$\Rightarrow \text{solution of } \max_{q_1} q_1(\alpha - c - q_1 - q_2)$$

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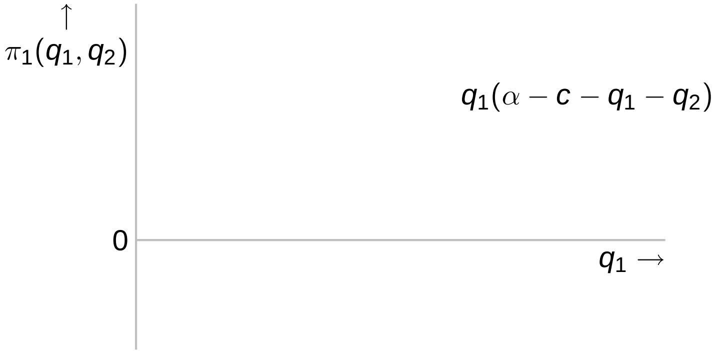
Nash equilibrium



Firm 1's payoff as a function of q_1, given q_2

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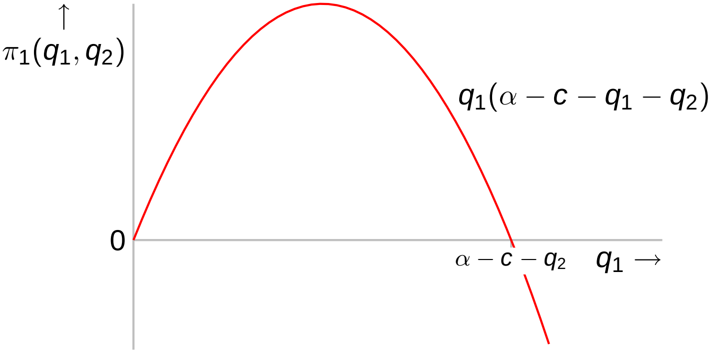

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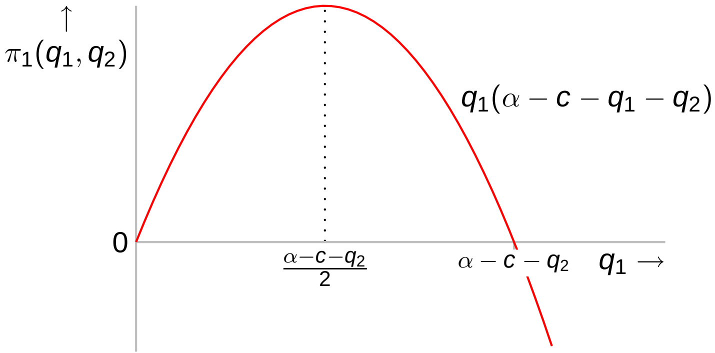

Firm 1's payoff as a function of q_1, given q_2

$\Rightarrow$ optimal q_1 given q_2 is $\frac{1}{2}(\alpha - c - q_2)$

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$$b_1(q_2) = \frac{1}{2}(\alpha - c - q_2)$$

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or

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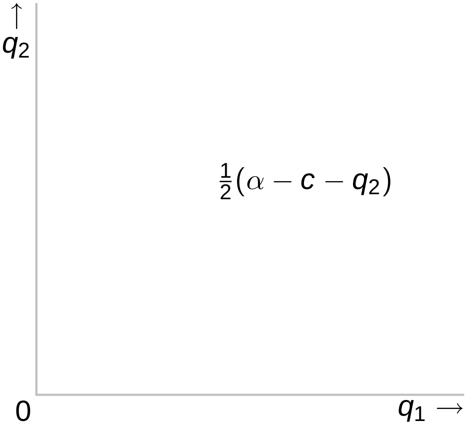

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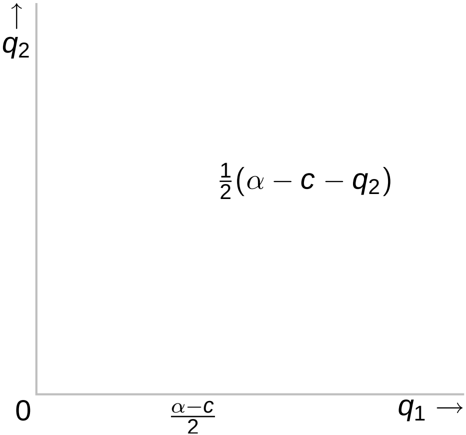

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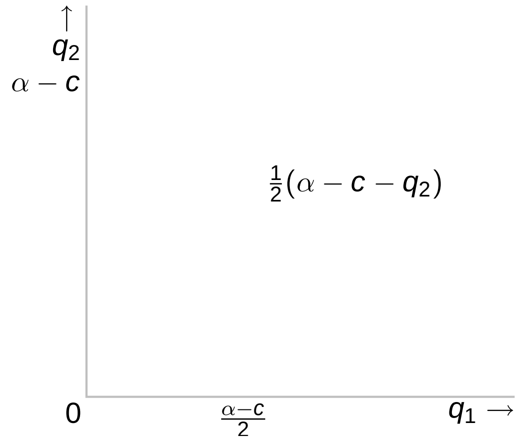

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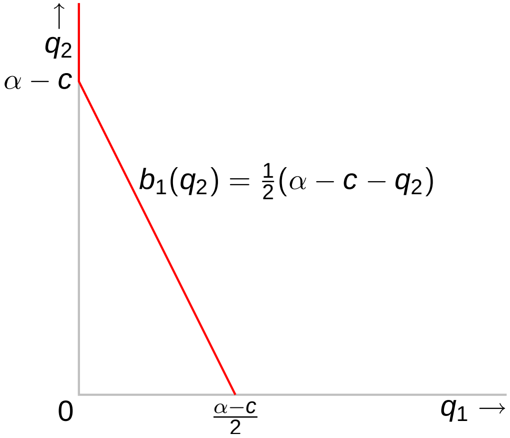
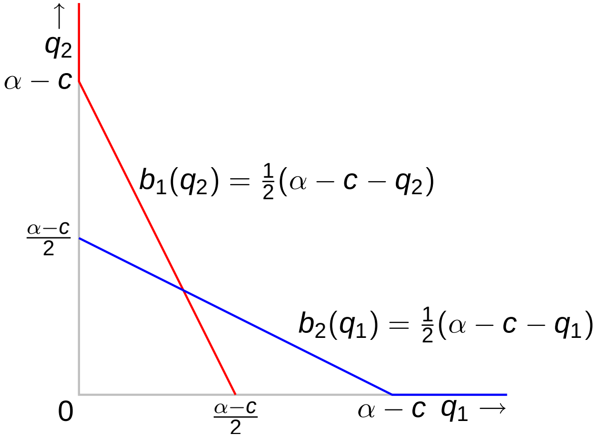


Example of Cournot's game

Nash equilibrium

$$q_1^* = \frac{1}{2}(\alpha - c - q_2^*) \; (= b_1(q_2^*))$$
$$q_2^* = \frac{1}{2}(\alpha - c - q_1^*) \; (= b_2(q_1^*))$$



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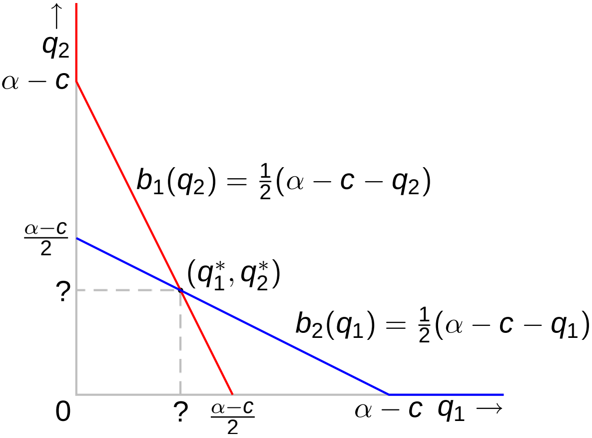

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Nash equilibrium

$$q_1^* = \frac{1}{2}(\alpha - c - q_2^*)$$
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Example of Cournot's game

Nash equilibrium

$$q_1^* = \frac{1}{2}(\alpha - c - q_2^*)$$
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$\Rightarrow$

$$q_1^* = \frac{1}{2}(\alpha - c - \frac{1}{2}(\alpha - c - q_1^*))$$

Example of Cournot's game

Nash equilibrium

$$q_1^* = \frac{1}{2}(\alpha - c - q_2^*)$$
$$q_2^* = \frac{1}{2}(\alpha - c - q_1^*)$$

$\Rightarrow$

$$q_1^* = \frac{1}{2}(\alpha - c - \frac{1}{2}(\alpha - c - q_1^*))$$
$$q_1^* = \frac{1}{4}(\alpha - c) + \frac{1}{4}q_1^*$$

Example of Cournot's game

Nash equilibrium

$$q_1^* = \frac{1}{2}(\alpha - c - q_2^*)$$
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$$\frac{3}{4}q_1^* = \frac{1}{4}(\alpha - c)$$

Example of Cournot's game

Nash equilibrium

$$q_1^* = \frac{1}{2}(\alpha - c - q_2^*)$$
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$$q_1^* = \frac{1}{3}(\alpha - c)$$

Example of Cournot's game

Nash equilibrium

$$q_1^* = \frac{1}{2}(\alpha - c - q_2^*)$$
$$q_2^* = \frac{1}{2}(\alpha - c - q_1^*)$$

$\Rightarrow$

$$q_1^* = \frac{1}{2}(\alpha - c - \frac{1}{2}(\alpha - c - q_1^*))$$
$$q_1^* = \frac{1}{4}(\alpha - c) + \frac{1}{4}q_1^*$$
$$\frac{3}{4}q_1^* = \frac{1}{4}(\alpha - c)$$
$$q_1^* = \frac{1}{3}(\alpha - c)$$

Substitute back to get $q_2^* = \frac{1}{2}(\alpha - c - \frac{1}{3}(\alpha - c)) = \frac{1}{3}(\alpha - c)$

Example of Cournot's game

Nash equilibrium

Conclusion: Unique Nash equilibrium,

$$(q_1^*, q_2^*) = (\tfrac{1}{3}(\alpha - c), \tfrac{1}{3}(\alpha - c))$$

Example of Cournot's game

Nash equilibrium

Conclusion: Unique Nash equilibrium,

$$(q_1^*, q_2^*) = (\tfrac{1}{3}(\alpha - c), \tfrac{1}{3}(\alpha - c))$$

Properties of Nash equilibrium

How is the price in the equilibrium related to unit cost, c?

Total output $=$

Example of Cournot's game

Nash equilibrium

Conclusion: Unique Nash equilibrium,

$$(q_1^*, q_2^*) = (\tfrac{1}{3}(\alpha - c), \tfrac{1}{3}(\alpha - c))$$

Properties of Nash equilibrium

How is the price in the equilibrium related to unit cost, c?

$$\text{Total output } = q_1^* + q_2^*$$

Example of Cournot's game

Nash equilibrium

Conclusion: Unique Nash equilibrium,

$$(q_1^*, q_2^*) = \left(\tfrac{1}{3}(\alpha - c), \tfrac{1}{3}(\alpha - c)\right)$$

Properties of Nash equilibrium

How is the price in the equilibrium related to unit cost, c?

$$\text{Total output } = q_1^* + q_2^* = \tfrac{1}{3}(\alpha - c) + \tfrac{1}{3}(\alpha - c)$$

Example of Cournot's game

Nash equilibrium

Conclusion: Unique Nash equilibrium,

$$(q_1^*, q_2^*) = (\tfrac{1}{3}(\alpha - c), \tfrac{1}{3}(\alpha - c))$$

Properties of Nash equilibrium

How is the price in the equilibrium related to unit cost, c?

$$\text{Total output } = q_1^* + q_2^* = \tfrac{1}{3}(\alpha - c) + \tfrac{1}{3}(\alpha - c) = \tfrac{2}{3}(\alpha - c)$$

Example of Cournot's game

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$$(q_1^*, q_2^*) = (\tfrac{1}{3}(\alpha - c), \tfrac{1}{3}(\alpha - c))$$

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How is the price in the equilibrium related to unit cost, c?

$$\text{Total output } = q_1^* + q_2^* = \tfrac{1}{3}(\alpha - c) + \tfrac{1}{3}(\alpha - c) = \tfrac{2}{3}(\alpha - c)$$

$$\Rightarrow \text{price } =$$

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Properties of Nash equilibrium

How is the price in the equilibrium related to unit cost, c?

$$\begin{aligned} \text{Total output } &= q_1^* + q_2^* = \tfrac{1}{3}(\alpha - c) + \tfrac{1}{3}(\alpha - c) = \tfrac{2}{3}(\alpha - c) \\ \Rightarrow \text{price } &= P(\tfrac{2}{3}(\alpha - c)) \end{aligned}$$

Example of Cournot's game

Nash equilibrium

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We have $\alpha > c$, so price $> c$

Example of Cournot's game

Nash equilibrium

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$$(q_1^*, q_2^*) = (\tfrac{1}{3}(\alpha - c), \tfrac{1}{3}(\alpha - c))$$

Properties of Nash equilibrium

How is the equilibrium related to the monopoly outcome?
Monopolist:

$$q^m \text{ solves } \max_q q(\alpha - q) - cq$$

Example of Cournot's game

Nash equilibrium

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$$\Rightarrow \text{total output in duopoly} > \text{monopoly output}$$

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$$\Rightarrow q^m = \tfrac{1}{2}(\alpha - c)$$

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$$\Rightarrow \text{price in duopoly } < \text{ monopoly price}$$

Example of Cournot's game: many firms

- Suppose number of firms is n, arbitrary number

Example of Cournot's game: many firms

- Suppose number of firms is n, arbitrary number
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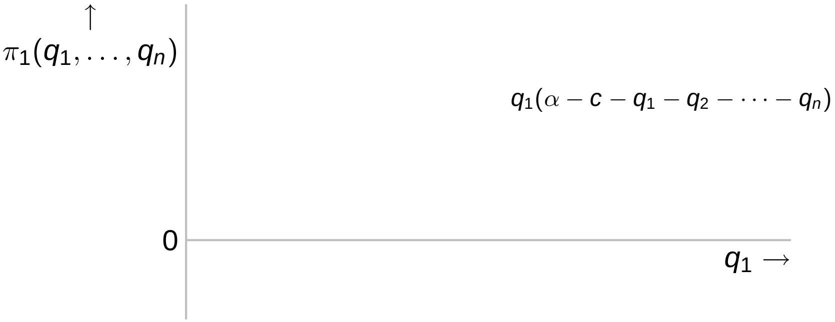

Firm 1's payoff as a function of q_1, given $q_2, \ldots, q_n$

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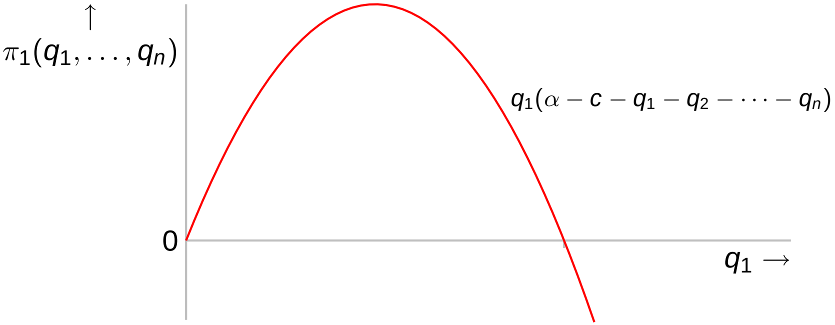

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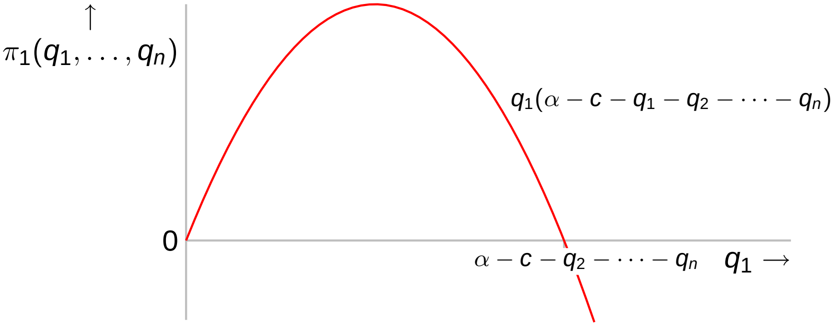

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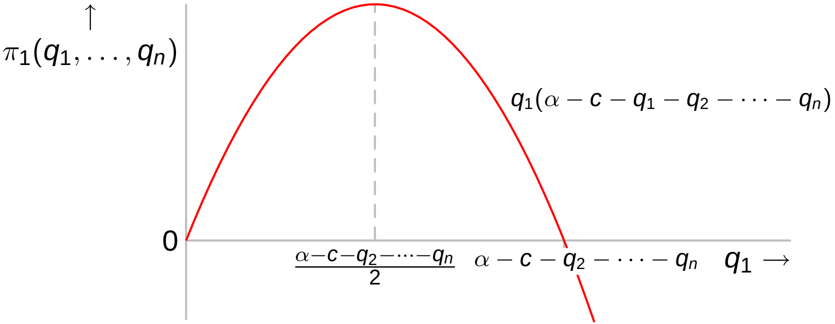

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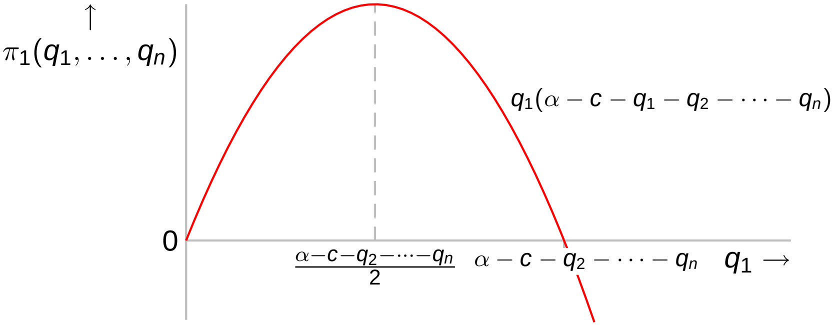

Firm 1's payoff as a function of q_1, given $q_2, \ldots, q_n$

$\Rightarrow$ optimal q_1 given $q_2, \ldots, q_n$ is $\frac{1}{2}(\alpha - c - q_2 - \cdots - q_n)$

Example of Cournot's game: many firms

- Best response function of firm 1 is

$$b_1(q_{-1}) = \frac{1}{2}(\alpha - c - q_2 - \cdots - q_n) \quad (\text{if } q_2 + \cdots + q_n \leq \alpha - c)$$

where q_{-1} stands for $(q_2, \ldots, q_n)$

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$$q_1^* = b_1(q_{-1}^*)$$

Example of Cournot's game: many firms

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Example of Cournot's game: many firms

- So for Nash equilibrium

$$
\begin{aligned}
q_1^* &= \tfrac{1}{2}(\alpha - c \qquad\quad - q_2^* - q_3^* - \cdots - q_n^*) \\
q_2^* &= \tfrac{1}{2}(\alpha - c - q_1^* \qquad\quad - q_3^* - \cdots - q_n^*) \\
&\vdots \\
q_n^* &= \tfrac{1}{2}(\alpha - c - q_1^* - q_2^* - \cdots - q_{n-1}^* \qquad)
\end{aligned}
$$

Example of Cournot's game: many firms

- So for Nash equilibrium

$$\begin{aligned} q_1^* &= \tfrac{1}{2}(\alpha - c - q_2^* - q_3^* - \cdots - q_n^*) \\ q_2^* &= \tfrac{1}{2}(\alpha - c - q_1^* - q_3^* - \cdots - q_n^*) \\ &\vdots \\ q_n^* &= \tfrac{1}{2}(\alpha - c - q_1^* - q_2^* - \cdots - q_{n-1}^*) \end{aligned}$$

- Multiply each equation by 2:

$$\begin{aligned} 2q_1^* &= \alpha - c - q_2^* - q_3^* - \cdots - q_n^* \\ 2q_2^* &= \alpha - c - q_1^* - q_3^* - \cdots - q_n^* \\ &\vdots \\ 2q_n^* &= \alpha - c - q_1^* - q_2^* - \cdots - q_{n-1}^* \end{aligned}$$

Example of Cournot's game: many firms

- From previous slide:

$$
\begin{aligned}
2q_1^* &= \alpha - c \qquad\quad - q_2^* - q_3^* - \cdots - q_n^* \\
2q_2^* &= \alpha - c - q_1^* \qquad\quad - q_3^* - \cdots - q_n^* \\
&\vdots \\
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$$

Example of Cournot's game: many firms

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$$
\begin{aligned}
2q_1^* &= \alpha - c - q_2^* - q_3^* - \cdots - q_n^* \\
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$$

- Subtract q_i^* from both sides of each equation i:

$$
\begin{aligned}
q_1^* &= \alpha - c - q_1^* - q_2^* - q_3^* - \cdots - q_n^* \\
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&\vdots \\
q_n^* &= \alpha - c - q_1^* - q_2^* - q_3^* - \cdots - q_n^*
\end{aligned}
$$

- Right-hand side of every equation is the same! So

$$q_1^* = q_2^* = \cdots = q_n^*$$

Example of Cournot's game: many firms

- Let $q_1^* = q_2^* = q_3^* = q_4^* = \cdots = q_n^* = q^*$

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(or any of the other equations)

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$$(n+1)q^* = \alpha - c$$

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  (or any of the other equations)
- Result is

$$(n+1)q^* = \alpha - c$$

- So

$$q^* = \frac{\alpha - c}{n+1}$$

Example of Cournot's game: many firms

Conclusion

- Game has unique Nash equilibrium, in which output of every firm i is

$$\frac{\alpha - c}{n + 1}$$

Example of Cournot's game: many firms

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- Game has unique Nash equilibrium, in which output of every firm i is
$$\frac{\alpha - c}{n + 1}$$
- Total output is $n(\alpha - c)/(n + 1)$

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- Game has unique Nash equilibrium, in which output of every firm i is
$$\frac{\alpha - c}{n + 1}$$
- Total output is $n(\alpha - c)/(n + 1)$
- Price is $\alpha - n(\alpha - c)/(n + 1)$, or
$$\frac{\alpha + nc}{n + 1}$$

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$$\frac{\alpha + nc}{n+1} = \frac{\alpha - c + (n+1)c}{n+1}$$

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- As n increases, this price decreases to c
- As number of firms increases, equilibrium outcome approaches competitive outcome

Comparison of Bertrand's and Cournot's games

Bertrand

- strategic variable is price

Cournot

Comparison of Bertrand's and Cournot's games

Bertrand

- strategic variable is price
- firm changes behavior if profit ↑ assuming other prices don't change (output adjusts)

Cournot

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- strategic variable is output

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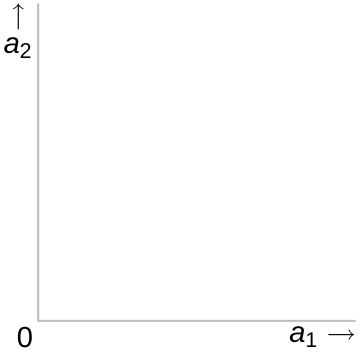

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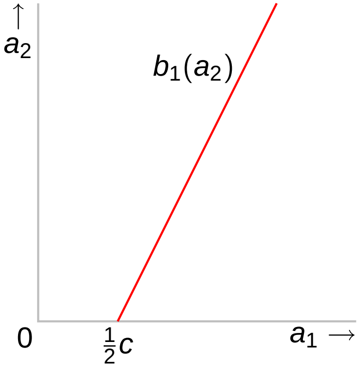

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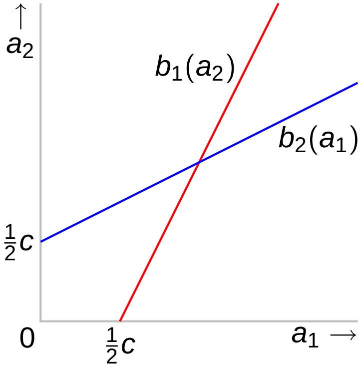

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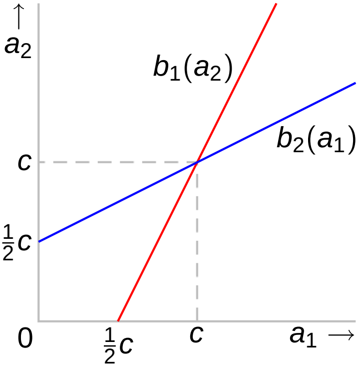

Unique Nash equilibrium: $(a_1^*, a_2^*) = (c, c)$

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Algebraically:

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   - Can in principle be used for any game, but computing complete best response functions may be difficult and unnecessary (as for Bertrand's game)