Economics 316

Fall 2017 Martin J. Osborne

Solutions to Problem Set 1

1. The first game is the same as *Bach or Stravinsky?* The reason is that the players' *preferences* are the same and their preferences in *Bach or Stravinsky?*, even though the payoff representations of these preferences are not the same. Specifically, if we associate X with *Bach* and Y with *Stravinsky*, then in both cases player 1's preferences are $(X, X) \succ (Y, Y) \succ (X, Y) \sim (Y, X)$ and player 2's preferences are $(Y, Y) \succ (X, X) \succ (X, Y) \sim (Y, X)$.

   The second game is not the same as *Bach or Stravinsky?* Again associating X with *Bach* and Y with *Stravinsky*, in *Bach or Stravinsky?* player 1 is indifferent between (X, Y) and (Y, X) whereas in the game in the question she is not (and neither is player 2).

2. (a) is not correct: the fact that both players are better off at (B, R) than they are at (T, L) is not relevant to the status of (T, L) as a Nash equilibrium.

   (b) is correct: the fact that player 1 prefers B to T when player 2 chooses L means that (T, L) is not a Nash equilibrium.

   (c) is correct: the fact that player 2 prefers R to L when player 1 chooses T means that (T, L) is not a Nash equilibrium.

   (d) is not correct: (T, L) is not a Nash equilibrium.

   (e) is not correct: although player 2 is worse off if she deviates, player 1 is better off, so that (B, R) is not a Nash equilibrium.

   (f) is correct: (B, R) is not a Nash equilibrium because player 1 is better off if she deviates to T.

3. The following game models the situation.

   Players The n people.

   Actions Each person's set of actions is $\{\textit{Contribute}, \textit{Don't contribute}\}$.

   Preferences Each person's preferences are those given in the problem.

An action profile in which more than k people contribute is not a Nash equilibrium: any contributor can induce an outcome she prefers by deviating to not contributing.

An action profile in which k people contribute is a Nash equilibrium: if any contributor stops contributing then the good is not provided; if any noncontributor switches to contributing then she is worse off.

An action profile in which fewer than k people contribute is a Nash equilibrium only if no one contributes: if someone contributes, she can increase her payoff by switching to noncontribution.

In summary, the set of Nash equilibria is the set of action profiles in which k people contribute together with the action profile in which no one contributes.

4. (a) The set of players consists of the n people, and each player has two possible actions, X and Y. Each player's preferences are represented by the payoff function that assigns her travel time to each action profile.

   (b) An action profile is a Nash equilibrium of the game if and only if either $n/2$ players choose X and $n/2$ choose Y, or $n/2+1$ choose X and $n/2-1$ choose Y.

   Here is the argument. Suppose that k players choose X and $n-k$ choose Y. Then the travel time of each player who chooses X is $k+1$ and the travel time of each player who chooses Y is $n-k+2$.

   Suppose $k = n/2$. Then the travel time of every player who chooses X is $n/2+1$ and the travel time of every player who chooses Y is $n/2+2$. If a player who chooses X switches to Y, there will be $n/2+1$ players choosing Y, so the player's travel time becomes $n/2+3$, which is larger than $n/2+1$. Thus she is worse off. If a player who chooses Y switches to X, there will be $n/2+1$ players choosing X, so the player's travel time becomes $n/2+2$. Thus her travel time remains the same. Hence any action profile with $k = n/2$ is a Nash equilibrium.

   Suppose $k = n/2+1$. Then if a player who chooses X switches to Y, her travel time remains $n/2+2$, so she is worse off. If a player who chooses Y switches to X, her travel time increases to $n/2+3$. Thus any action profile with $k = n/2+1$ a Nash equilibrium.

   Suppose $k \leq n/2-1$. If a player who chooses Y switches to X, her travel time becomes $k+2$, which is less than $n-k+2$ given that $k \leq n/2 - 1 < n/2$. Thus no action profile with $k \leq n/2 - 1$ a Nash equilibrium.

Suppose $k \geq n/2 + 2$. If a player who chooses X switches to Y, her travel time becomes $n - k + 3$, which is less than $k + 1$ given that $k \geq n/2 + 2 > n/2 + 1$. Thus no action profile with $k \geq n/2 + 2$ is a Nash equilibrium.

(c) Consider the outcome generated by an action profile in which every person takes the *same* route. This outcome is not Pareto efficient because in the action profile in which one person is moved to the other route everyone is better off.

Now consider an outcome generated by any other action profile. Moving one or more people from one route to the other increases the travel time of every person taking the second route, so the outcome is Pareto efficient.

Thus the Nash equilibria are Pareto efficient, but so are the outcomes generated by any other action profile in which at least one person takes each route.