Economics 316

Fall 2017 | Martin J. Osborne

Solutions to problems for Tutorial 1

1. (a) The situation is modeled by the following game.

	Passive	*Aggressive*
Passive	2, 2	1, 3
Aggressive	3, 1	0, 0

(The numbers 0, 1, 2, and 3 can be replaced by any other numbers with the same ordering—for example, −2, 6, 8, and 100.)

(b) The players' preferences differ from their preferences in the *Prisoner's Dilemma* because when one player chooses *Aggressive* the other player prefers *Passive* to *Aggressive* rather than *Aggressive* to *Passive*.

(c) The Nash equilibria of the game are (*Passive*, *Aggressive*) and (*Aggressive*, *Passive*).

2. (a) A game that models the situation is shown in Figure 1.

	Sit	*Stand*
Sit	1, 1	2, 0
Stand	0, 2	0, 0

Figure 1. Behavior on a bus when the players' preferences are selfish.

This game is not the *Prisoner's Dilemma*. If we identify *Sit* with *High* and *Stand* with *Low* then, for example, (*Stand*, *Sit*) is worse for player 1 than (*Sit*, *Sit*), rather than better. If we identify *Sit* with *Low* and *Stand* with *High* then, for example, (*Stand*, *Stand*) is worse for player 1 than (*Sit*, *Sit*), rather than better. The game has a unique Nash equilibrium, (*Sit*, *Sit*).

(b) A game that models the situation is shown in Figure 2.

This game is the *Prisoner's Dilemma*. Its unique Nash equilibrium is the action pair (*Stand*, *Stand*).

	Sit	*Stand*
Sit	2,2	0,3
Stand	3,0	1,1

Figure 2. Behavior on a bus when the players' preferences are altruistic.

(c) Both people are more comfortable in the equilibrium that results when they act according to their selfish preferences.

3. (a) For $k = m = 1$ the game is shown in Figure 3. It is the same, except for the names of the actions, as the *Prisoner's Dilemma*.

		B supporter	
		abstain	*vote*
A supporter	*abstain*	$1,1$	$0,2-c$
	vote	$2-c,0$	$1-c,1-c$

Figure 3. The game of voter participation in Problem 3.

(b) For $k = m$, denote the number of citizens voting for A by n_A and the number voting for B by n_B. The cases in which $n_A \leq n_B$ are symmetric with those in which $n_A \geq n_B$; I restrict attention to the latter.

$n_A = n_B = k$ (all citizens vote): A citizen who switches from voting to abstaining causes the candidate she supports to lose rather than tie, reducing her payoff from $1 - c$ to 0. Since $c < 1$, this situation is a Nash equilibrium.

$n_A = n_B < k$ (not all citizens vote; the candidates tie): A citizen who switches from abstaining to voting causes the candidate she supports to win rather than tie, increasing her payoff from 1 to $2 - c$. Thus this situation is not a Nash equilibrium.

$n_A = n_B + 1$ **or** $n_B = n_A + 1$ (a candidate wins by one vote): A supporter of the losing candidate who switches from abstaining to voting causes the candidate she supports to tie rather than lose, increasing her payoff from 0 to $1 - c$. Thus this situation is not a Nash equilibrium.

$n_A \geq n_B + 2$ **or** $n_B \geq n_A + 2$ (a candidate wins by two or more votes): A supporter of the winning candidate who switches from voting to abstaining does not affect the outcome, but saves the cost c, so such a situation is not a Nash equilibrium.

We conclude that the game has a unique Nash equilibrium, in which all citizens vote.

(c) If $k < m$ then a similar logic shows that there is no Nash equilibrium.

$n_A = n_B \leq k$**:** A supporter of B who switches from abstaining to voting causes B to win rather than tie, increasing her payoff from 1 to $2 - c$. Thus this situation is not a Nash equilibrium.

$n_A = n_B + 1$**, or** $n_B = n_A + 1$ **and** $n_A < k$**:** A supporter of the losing candidate who switches from abstaining to voting causes the candidates to tie, increasing her payoff from 0 to $1 - c$. Thus this situation is not a Nash equilibrium.

$n_A = k$ **and** $n_B = k + 1$**:** A supporter of the losing candidate (namely A) who switches from voting to abstaining does not affect the outcome but saves the cost c. Thus this situation is not a Nash equilibrium.

$n_A \geq n_B + 2$ **or** $n_B \geq n_A + 2$**:** Any citizen who switches from voting to abstaining does not affect the outcome but saves the cost c, so such a situation is not a Nash equilibrium.