# ECO316: Applied game theory

## Lecture 1

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# Policy on use of electronic devices

- No electronic devices may be used *except* in the first 5 rows of the classroom
- In the first 5 rows, electronic devices may be use *only* to view the class slides and take notes

# Game theory

- Analytical tools for studying situations in which decision-makers interact
- Used in economics, psychology, political science, sociology, computer science, biology
- Course covers basic theory, with emphasis on applications in economics
  - ECO326 is more formal, more suitable prep for theoretically-oriented grad program

# Economic theory

1. Economic question
   - More firms $\Rightarrow$ better outcome for consumers?
   - How do political parties' positions depend on voters' preferences in a first-past-the-post election?
   - Is unanimity a good voting rule for a jury?
2. Formulate model that captures essence of situation
   - In this course: game-theoretic model
3. Analyze model using appropriate tools
   - Typically look for an "equilibrium"
4. Extract from analysis insights about economic question

# Applications: preview

- Competition between firms
  - More firms $\Rightarrow$ better outcome for consumers?
- Electoral competition
  - How well do electoral outcomes reflect votes' preferences?
- Markets with asymmetric information
  - Can outcome be improved by regulation?
- Voting
  - How does outcome depend on voting rule?
  - Is unanimity a good voting rule for an jury?
- Auctions
  - Which design produces an efficient outcome? High revenue for the auctioneer?
- Matching
  - Which algorithm produces a good outcome?

# Voting in a jury

- Jury is presented with evidence
- Members vote
- Unanimity rule: defendant is convicted only if every member votes to convict

# Voting in a jury

- When does your vote make a difference to the decision?

# Voting in a jury

- When does your vote make a difference to the decision?
- Not if some other jurors vote to acquit . . .


- . . . only if *all* other jurors vote to convict

# Voting in a jury


- What should a juror infer if all other jurors vote to convict?
- That all jurors find the evidence of guilt very compelling?
- So a juror should vote to convict even if her own personal evaluation of the evidence favors conviction only mildly?
- But all the other jurors should reason in the same way
- . . . so the nature of an equilibrium is unclear
- General point: decision-maker may need to consider other decision-makers' *information* when choosing her action

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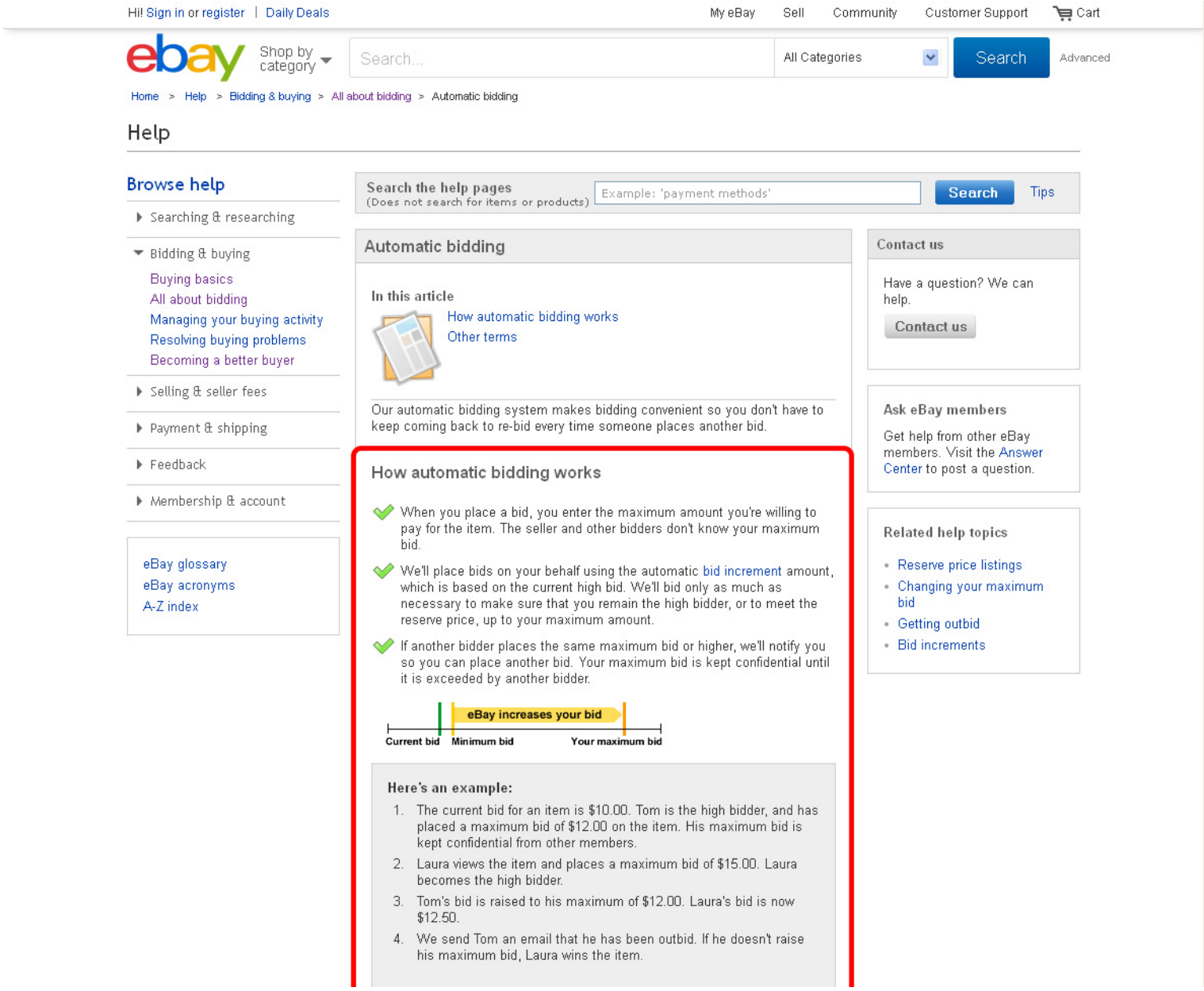
# Auctions


Hi! Sign in or register | Daily Deals
My eBay Sell Community Customer Support Cart
ebay Shop by category
Search...
All Categories
Search
Advanced
Home > Help > Bidding & buying > All about bidding > Automatic bidding
Help
Browse help
Searching & researching
Bidding & buying
Buying basics
All about bidding
Managing your buying activity
Resolving buying problems
Becoming a better buyer
Selling & seller fees
Payment & shipping
Feedback
Membership & account
eBay glossary
eBay acronyms
A-Z index
Search the help pages
(Does not search for items or products)
Example: 'payment methods'
Search
Tips
Automatic bidding
In this article
How automatic bidding works
Other terms
Our automatic bidding system makes bidding convenient so you don't have to keep coming back to re-bid every time someone places another bid.
How automatic bidding works
When you place a bid, you enter the maximum amount you're willing to pay for the item. The seller and other bidders don't know your maximum bid.
We'll place bids on your behalf using the automatic bid increment amount, which is based on the current high bid. We'll bid only as much as necessary to make sure that you remain the high bidder, or to meet the reserve price, up to your maximum amount.
If another bidder places the same maximum bid or higher, we'll notify you so you can place another bid. Your maximum bid is kept confidential until it is exceeded by another bidder.
eBay increases your bid
Current bid
Minimum bid
Your maximum bid
Here's an example:
1. The current bid for an item is $10.00. Tom is the high bidder, and has placed a maximum bid of $12.00 on the item. His maximum bid is kept confidential from other members.
2. Laura views the item and places a maximum bid of $15.00. Laura becomes the high bidder.
3. Tom's bid is raised to his maximum of $12.00. Laura's bid is now $12.50.
4. We send Tom an email that he has been outbid. If he doesn't raise his maximum bid, Laura wins the item.
Contact us
Have a question? We can help.
Contact us
Ask eBay members
Get help from other eBay members. Visit the Answer Center to post a question.
Related help topics
Reserve price listings
Changing your maximum bid
Getting outbid
Bid increments

# Auctions

## How automatic bidding works

- When you place a bid, you enter the maximum amount you're willing to pay for the item. The seller and other bidders don't know your maximum bid.
- We'll place bids on your behalf using the automatic bid increment amount, which is based on the current high bid. We'll bid only as much as necessary to make sure that you remain the high bidder, or to meet the reserve price, up to your maximum amount.
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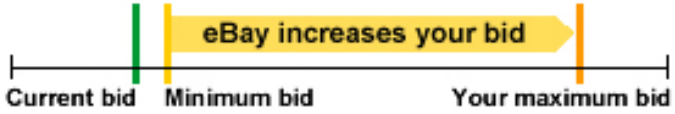


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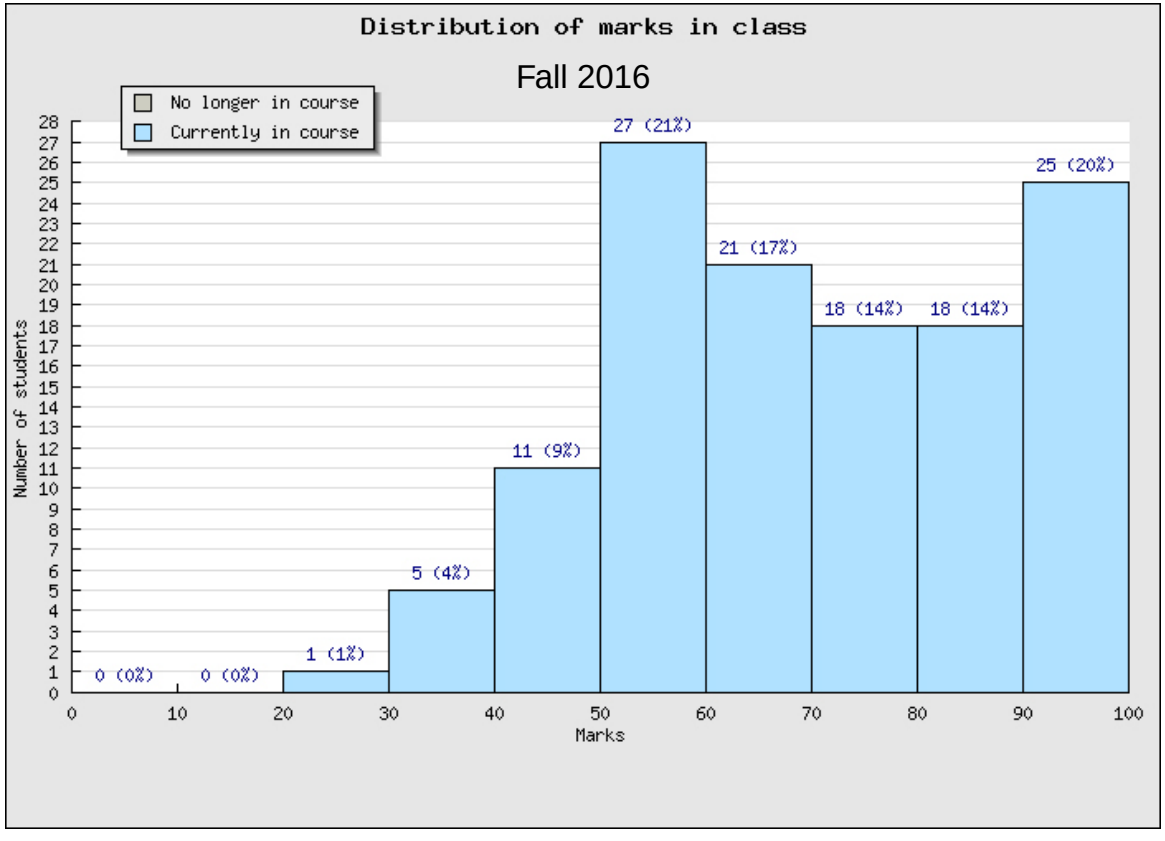
# Course website

http://mjo.osborne.economics.utoronto.ca/index.php/course/index/9

Username: *UTORid*

Password: *U of T student number (without leading 0)*



# Rational decision-maker

- Will concentrate on models in which decision-makers are rational
- Rational means
  - well-defined preferences
  - actions are best according to preferences, given constraints
- Rational does *not* mean
  - fully informed
  - selfish
  - sensible in an objective sense

# Rational decision-maker

## Model

A decision problem consists of

- a set $A$ of possible *actions*
- *preferences* over $A$

## Theory

Decision-maker chooses the member of $A$ that is best according to her preferences

# Many decision-makers: Strategic games

- Game theory concerns situations in which decision-makers *interact*
- Simple model of interaction is strategic game

A strategic game consists of

- a set of *players*
- for each player
  - a set of possible *actions*
  - *preferences* over the collection of action profiles

# Many decision-makers: Strategic games

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  - a set of possible *actions*
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# Strategic game: Example

- Players: two firms
- For each firm:
  - possible actions: *low price*, *high price*
  - preferences: for firm 1,

$$(low, high) \succ (high, high) \succ (low, low) \succ (high, low)$$

and symmetrically for firm 2

# Strategic game: Example

- Players: two firms
- For each firm:
  - possible actions: *low price*, *high price*
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$$(low, high) \succ (high, high) \succ (low, low) \succ (high, low)$$

and symmetrically for firm 2

## Comments

- Highly simplified model!
- But pattern of payoffs is possible
- (Other patterns are also possible)

# Strategic game: Example

- Players: two firms
- For each firm:
  - possible actions: *low price*, *high price*
  - preferences: for firm 1,

$$(low, high) \succ (high, high) \succ (low, low) \succ (high, low)$$

and symmetrically for firm 2

## Working with preferences

Use numbers to represent them:

Firm 1's payoffs

| Firm 1 \ Firm 2 | *high* | *low* |
|---|---|---|
| *high* | 2 | 0 |
| *low* | 3 | 1 |

Firm 2's payoffs

| Firm 1 \ Firm 2 | *high* | *low* |
|---|---|---|
| *high* | 2 | 3 |
| *low* | 0 | 1 |

# Strategic game: Example

- Players: two firms
- For each firm:
  - possible actions: *low price*, *high price*
  - preferences: for firm 1,

$$(low, high) \succ (high, high) \succ (low, low) \succ (high, low)$$

and symmetrically for firm 2

## Working with preferences

Combine tables:

| | | Firm 2: *high* | Firm 2: *low* |
|---|---|---|---|
| Firm 1 | *high* | 2, 2 | 0, 3 |
| | *low* | 3, 0 | 1, 1 |

Payoff of firm 1, payoff of firm 2

# Strategic game: Example

| | | Firm 2 | |
|---|---|---|---|
| | | *high* | *low* |
| Firm 1 | *high* | 2, 2 | 0, 3 |
| | *low* | 3, 0 | 1, 1 |

## Notes

- We could use other numbers to represent preferences (e.g. profits)
- For current purposes, only *order* matters
- What defines game are players' *preferences*, not specific payoff representations

# Strategic game: Example

| | | Firm 2 | |
|---|---|---|---|
| | | *high* | *low* |
| Firm 1 | *high* | $2, 2$ | $0, 3$ |
| | *low* | $3, 0$ | $1, 1$ |

*Same* game:

| | | Firm 2 | |
|---|---|---|---|
| | | *high* | *low* |
| Firm 1 | *high* | $100, 100$ | $-5, 101$ |
| | *low* | $101, -5$ | $0, 0$ |

| | | Firm 2 | |
|---|---|---|---|
| | | *high* | *low* |
| Firm 1 | *high* | $0, 90$ | $-100, 100$ |
| | *low* | $100, -5$ | $-1, 1$ |

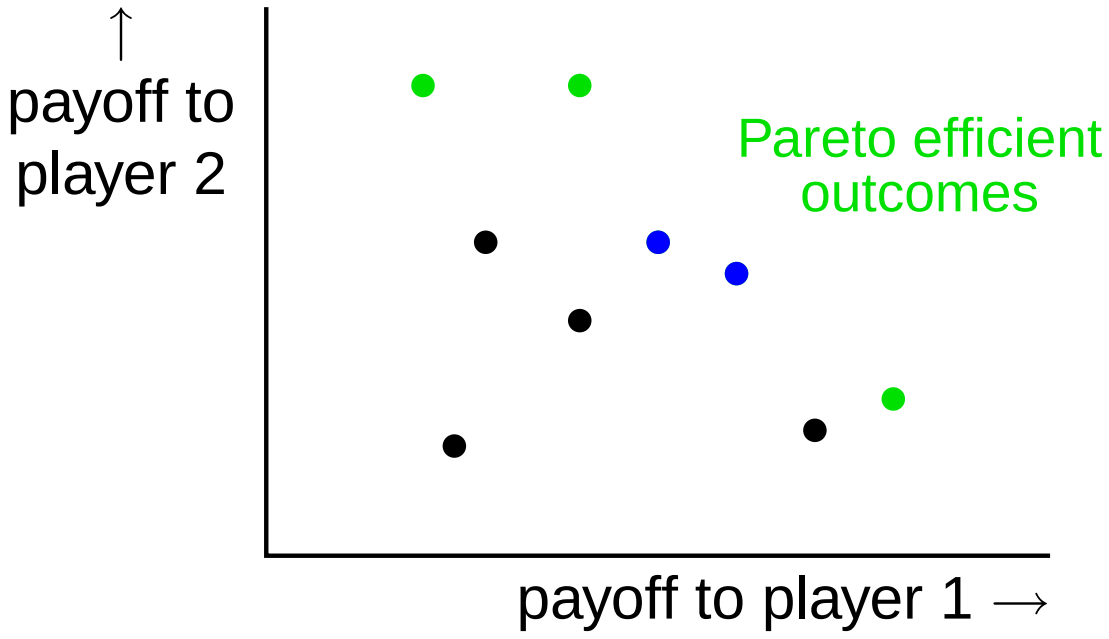
A game is determined by the players' *orderings* of the outcomes

# Strategic game: Example

| | | Firm 2 | |
|---|---|---|---|
| | | *high* | *low* |
| Firm 1 | *high* | 2, 2 | 0, 3 |
| | *low* | 3, 0 | 1, 1 |

- Game is called *Prisoner's Dilemma*
- Structure of incentives in game is present in many situations
- Has been used to model a huge variety of situations in diverse fields

# Digression: Pareto efficiency




- Which outcomes are Pareto efficient?
- An outcome *a* is (weakly) *Pareto efficient* if there is no outcome that *every* player prefers to *a*

# Strategic game: Prisoner's Dilemma

| | | Firm 2 | |
|---|---|---|---|
| | | *high* | *low* |
| Firm 1 | *high* | 2, 2 | 0, 3 |
| | *low* | 3, 0 | 1, 1 |

- Which outcomes (action pairs) are Pareto efficient?
- $(high, high)$, $(high, low)$, $(low, high)$

# Strategic games: More examples

In other examples, the pattern of incentives is different

| | $X$ | $Y$ |
|---|---|---|
| $X$ | $2, 2$ | $1, 1$ |
| $Y$ | $1, 1$ | $0, 0$ |

| | $X$ | $Y$ |
|---|---|---|
| $X$ | $2, 2$ | $0, 0$ |
| $Y$ | $0, 0$ | $1, 1$ |

Coordination game

| | $B$ | $S$ |
|---|---|---|
| $B$ | $2, 1$ | $0, 0$ |
| $S$ | $0, 0$ | $1, 2$ |

*Bach or Stravinsky?*

| | $X$ | $Y$ |
|---|---|---|
| $X$ | $1, -1$ | $-1, 1$ |
| $Y$ | $-1, 1$ | $1, -1$ |

*Matching pennies*

# A richer example

In examples so far, only two players, each with only two actions

## Cournot's oligopoly game

- Players: *n* firms
- For each firm
  - possible actions: outputs (nonnegative numbers)
  - payoff: profit

Notes

- Many players, each with continuum of actions
- *Cannot* represent game in a table

# Equilibrium outcomes

- We want to assume each player is rational
- But each player doesn't know what others will do—so how to choose action?
- Form *beliefs* about others' actions
- Where do beliefs come from?
- Assume players have experience playing the game, or similar games—in fact, assume that their beliefs are *correct*

# Equilibrium outcomes

- Each player's action is optimal given her beliefs
- Each player's belief is correct

$\Rightarrow$ each player's action is optimal given other players' actions

**Definition**

A Nash equilibrium of a strategic game is an action profile with the property that every player's action is optimal, given the other players' actions.

Equivalently: an action profile is a Nash equilibrium if no player can increase her payoff by changing her action, given the other players' actions

# Prisoner's Dilemma

| | | Firm 2 | |
|---|---|---|---|
| | | *high* | *low* |
| Firm 1 | *high* | $2, 2$ | $0, 3$ |
| | *low* | $3, 0$ | $1, 1$ |

Check each action pair in turn

- $(high, high)$: not a Nash equilibrium because firm 1 is better off deviating to *low* (and firm 2 is also better off deviating to *low*)
- $(high, low)$: not a Nash equilibrium because ....
- $(low, high)$: not a Nash equilibrium because ....
- $(low, low)$: Nash equilibrium because each player is worse off switching to *high* if other player's action is *low*.

So: unique Nash equilibrium, $(low, low)$.

# Prisoner's Dilemma

| | | Firm 2 | |
|---|---|---|---|
| | | *high* | *low* |
| Firm 1 | *high* | 2, 2 | 0, 3 |
| | *low* | 3, 0 | 1, 1 |

- Which outcomes (action pairs) are Pareto efficient?
  - $(low, high)$, $(high, high)$, $(high, low)$
- Note that the unique Nash equilibrium, $(low, low)$, is *not* Pareto efficient

# Split or steal?

| | *Split* | *Steal* |
|---|---|---|
| *Split* | 50,075, 50,075 | 0, 100,150 |
| *Steal* | 100,150, 0 | 0, 0 |

Nash equilibria? $(Split, Steal)$, $(Steal, Split)$, and $(Steal, Steal)$

Alternative representation:

| | *Split* | *Steal* |
|---|---|---|
| *Split* | 1, 1 | 0, 2 |
| *Steal* | 2, 0 | 0, 0 |

Compare with *Prisoner's Dilemma*:

| | *X* | *Y* |
|---|---|---|
| *X* | 2, 2 | 0, 3 |
| *Y* | 3, 0 | 1, 1 |

Only difference between games: indicated preferences

# Example of Nash equilibrium: Coordination game

| | $X$ | $Y$ |
|---|---|---|
| $X$ | $2, 2$ | $0, 0$ |
| $Y$ | $0, 0$ | $1, 1$ |

Two Nash equilibria, $(X, X)$ and $(Y, Y)$

# Example of Nash equilibrium: *Bach or Stravinsky?*

| | *Bach* | *Stravinsky* |
|---|---|---|
| *Bach* | $2, 1$ | $0, 0$ |
| *Stravinsky* | $0, 0$ | $1, 2$ |

Two Nash equilibria, $(B, B)$ and $(S, S)$

# Example of Nash equilibrium: *Matching Pennies*

| | $H$ | $T$ |
|---|---|---|
| $H$ | $1, -1$ | $-1, 1$ |
| $T$ | $-1, 1$ | $1, -1$ |

No Nash equilibrium!

# Example of Nash equilibrium

| | $L$ | $R$ |
|---|---|---|
| $T$ | $1, 1$ | $2, 1$ |
| $B$ | $0, 0$ | $2, 4$ |

Nash equilibria: $(T, L)$, $(T, R)$, and $(B, R)$

# Example: Investing in a joint project

- $n$ people
- Each person chooses whether to invest
- If at least $k$ people invest, project succeeds (where $k$ is a fixed number with $2 \leq k \leq n-1$)
- If fewer than $k$ people invest, project fails
- Project succeeds $\Rightarrow$ every investor gets positive return
- Project fails $\Rightarrow$ every investor suffers a loss
- Noninvestors unaffected by outcome of project
- So for every person,

  invest & project succeeds $\succ$ don't invest $\succ$ invest & project fails

# Investing in a joint project

## Strategic game

- Players: $n$ people
- For each player,
  - possible actions: *Invest*, *Don't invest*
  - payoffs: If player chooses *Invest*,

$$\begin{cases} 100 & \text{if at least } k \text{ people choose } \textit{Invest} \\ -10 & \text{if fewer than } k \text{ people choose } \textit{Invest} \end{cases}$$

If player chooses *Don't invest*, 0 regardless of others' actions

# Investing in a joint project

## Nash equilibrium

- $k$ people invest?
- $n$ people invest?
- no one invests?
- some other number of people invest?

# Investing in a joint project

Reminder of payoffs:

- *Invest* $\Rightarrow$ 100 if $\geq k$ investors, $-10$ if $< k$ investors
- *Don't invest* $\Rightarrow$ 0

## Nash equilibrium

- $n$ people invest: Nash equilibrium because player deviates $\Rightarrow$ gets 0 rather than 100
- no one invests: Nash equilibrium because player deviates $\Rightarrow$ gets $-10$ rather than 0
- between 1 and $k-1$ people invest: not Nash equilibrium because investor deviates $\Rightarrow$ gets 0 rather than $-10$
- between $k$ and $n-1$ people invest: not Nash equilibrium because noninvestor deviates $\Rightarrow$ gets 100 rather than 0

# Investing in a joint project

Reminder of payoffs:

- *Invest* $\Rightarrow$ 100 if $\geq k$ investors, $-10$ if $< k$ investors
- *Don't invest* $\Rightarrow$ 0

## Summary

Exactly two Nash equilibria:

- everyone invests
- no one invests

# Traveler's Dilemma

- Airline has lost suitcases of two travelers
- Suitcases and contents are identical
- Airline's process:
  - each traveler specifies value of her suitcase, a number from \$2 to \$100
  - if both travelers specify same number, they are paid that amount
  - if travelers specify different amounts,
    - traveler specifying smaller amount is paid *that amount plus* \$2
    - traveler specifying larger amount is paid *the smaller amount minus* \$2

# Traveler's Dilemma

## Strategic game

- Players: two travelers
- For each player,
  - possible actions: \$2, \$3, . . . , \$100
  - payoffs: for player $i$,

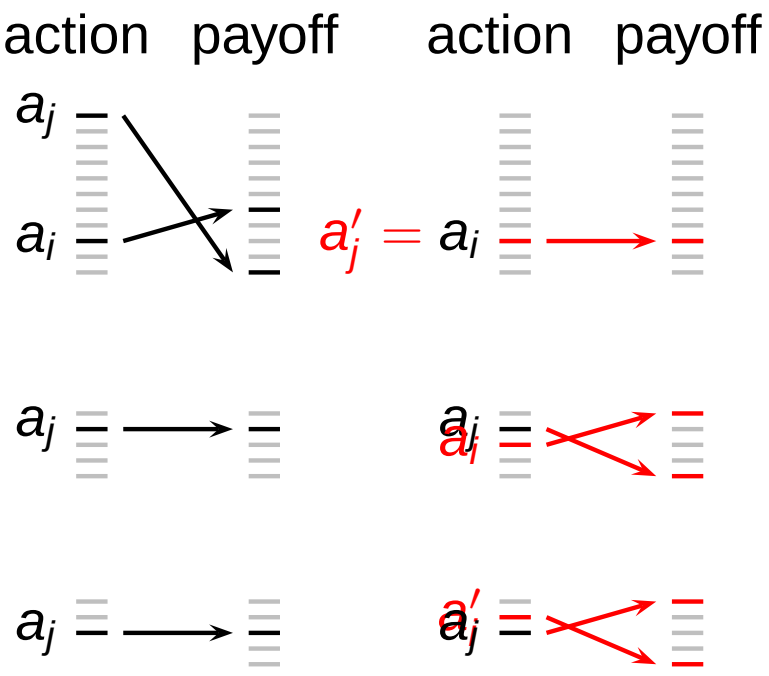
$$\begin{cases} a_i + 2 & \text{if } a_i < a_j \\ a_i & \text{if } a_i = a_j \\ a_j - 2 & \text{if } a_i > a_j \end{cases}$$

where $a_i$ is $i$'s action and $a_j$ is the other player's action

# Traveler's Dilemma

## Nash equilibrium

- $a_i < a_j$?
  - Not NE: $j$ lowers $a_j$ to $a_i$ $\Rightarrow$ increases $j$'s payoff
- $a_i = a_j$?
  - If $a_i \geq 3$, not NE: $i$ lowers $a_i$ to $a_i - 1 \Rightarrow$ increases $i$'s payoff
  - If $a_i = a_j = 2$, NE! If either player increases amount, payoff $= 0$



# Traveler's Dilemma

## Summary

Unique Nash equilibrium: both travelers name the lowest possible valuation, $2