ECO316: Applied game theory

Lecture 1

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Policy on use of electronic devices

- No electronic devices may be used *except* in the first 5 rows of the classroom
- In the first 5 rows, electronic devices may be use *only* to view the class slides and take notes

Game theory

- Analytical tools for studying situations in which decision-makers interact

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individual humans, groups of humans, animals, . . .

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  - ECO326 is more formal, more suitable prep for theoretically-oriented grad program

Economic theory

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4. Extract from analysis insights about economic question

Applications: preview

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Voting in a jury

- Jury is presented with evidence

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- Members vote

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- Members vote
- Unanimity rule: defendant is convicted only if every member votes to convict

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- But all the other jurors should reason in the same way
- . . . so the nature of an equilibrium is unclear
- General point: decision-maker may need to consider other decision-makers' *information* when choosing her action

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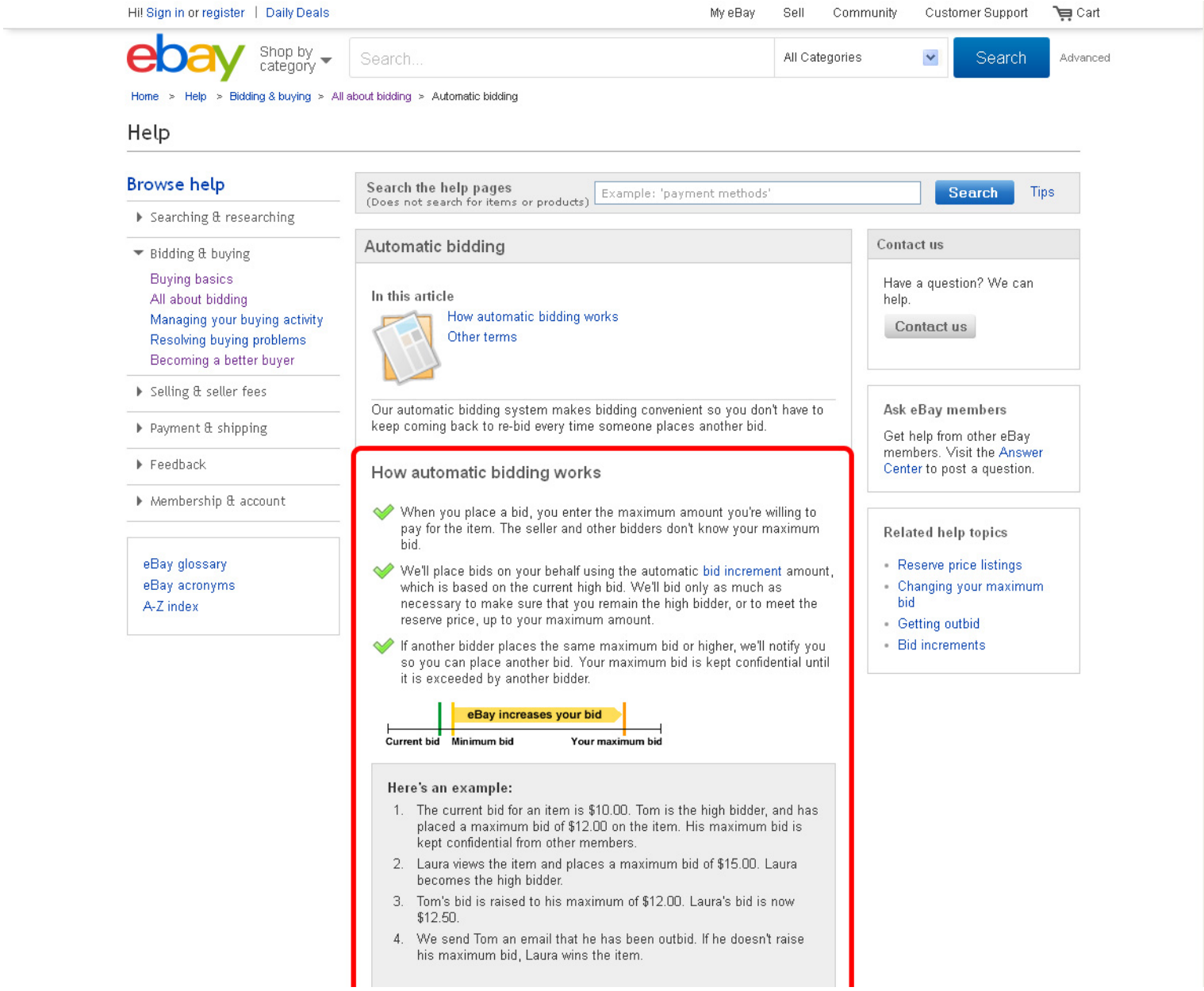
  - Which design produces an efficient outcome? High revenue for the auctioneer?

Auctions


Hi! Sign in or register | Daily Deals
My eBay Sell Community Customer Support Cart
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Search...
All Categories
Search
Advanced
Home > Help > Bidding & buying > All about bidding > Automatic bidding
Help
Browse help
Searching & researching
Bidding & buying
Buying basics
All about bidding
Managing your buying activity
Resolving buying problems
Becoming a better buyer
Selling & seller fees
Payment & shipping
Feedback
Membership & account
eBay glossary
eBay acronyms
A-Z index
Search the help pages
(Does not search for items or products)
Example: 'payment methods'
Search
Tips
Automatic bidding
In this article
How automatic bidding works
Other terms
Our automatic bidding system makes bidding convenient so you don't have to keep coming back to re-bid every time someone places another bid.
How automatic bidding works
When you place a bid, you enter the maximum amount you're willing to pay for the item. The seller and other bidders don't know your maximum bid.
We'll place bids on your behalf using the automatic bid increment amount, which is based on the current high bid. We'll bid only as much as necessary to make sure that you remain the high bidder, or to meet the reserve price, up to your maximum amount.
If another bidder places the same maximum bid or higher, we'll notify you so you can place another bid. Your maximum bid is kept confidential until it is exceeded by another bidder.
eBay increases your bid
Current bid
Minimum bid
Your maximum bid
Here's an example:
1. The current bid for an item is $10.00. Tom is the high bidder, and has placed a maximum bid of $12.00 on the item. His maximum bid is kept confidential from other members.
2. Laura views the item and places a maximum bid of $15.00. Laura becomes the high bidder.
3. Tom's bid is raised to his maximum of $12.00. Laura's bid is now $12.50.
4. We send Tom an email that he has been outbid. If he doesn't raise his maximum bid, Laura wins the item.
Contact us
Have a question? We can help.
Contact us
Ask eBay members
Get help from other eBay members. Visit the Answer Center to post a question.
Related help topics
Reserve price listings
Changing your maximum bid
Getting outbid
Bid increments

Auctions

How automatic bidding works

- When you place a bid, you enter the maximum amount you're willing to pay for the item. The seller and other bidders don't know your maximum bid.
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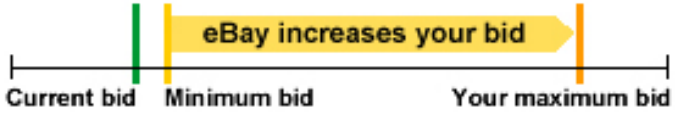

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- Matching
  - Which algorithm produces a good outcome?

Course website

http://mjo.osborne.economics.utoronto.ca/index.php/course/index/9

Username: *UTORid*
Password: *U of T student number (without leading 0)*

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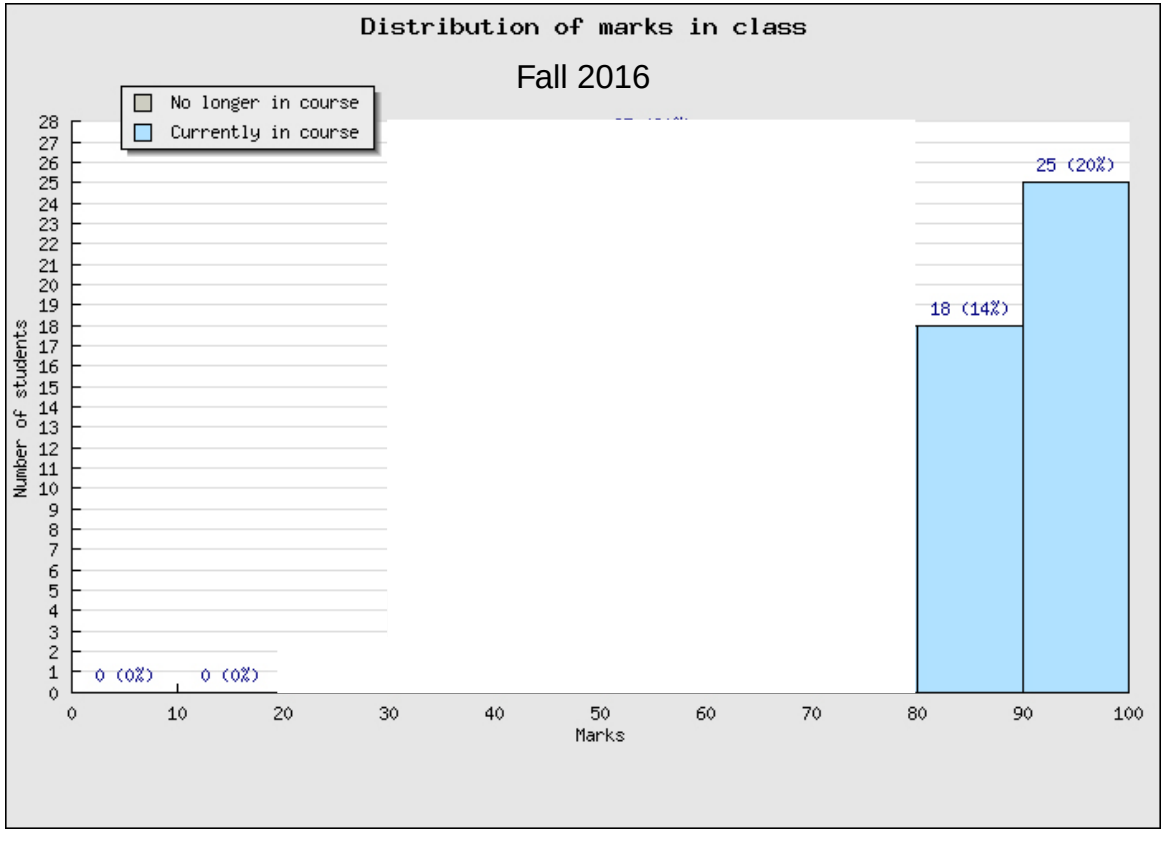

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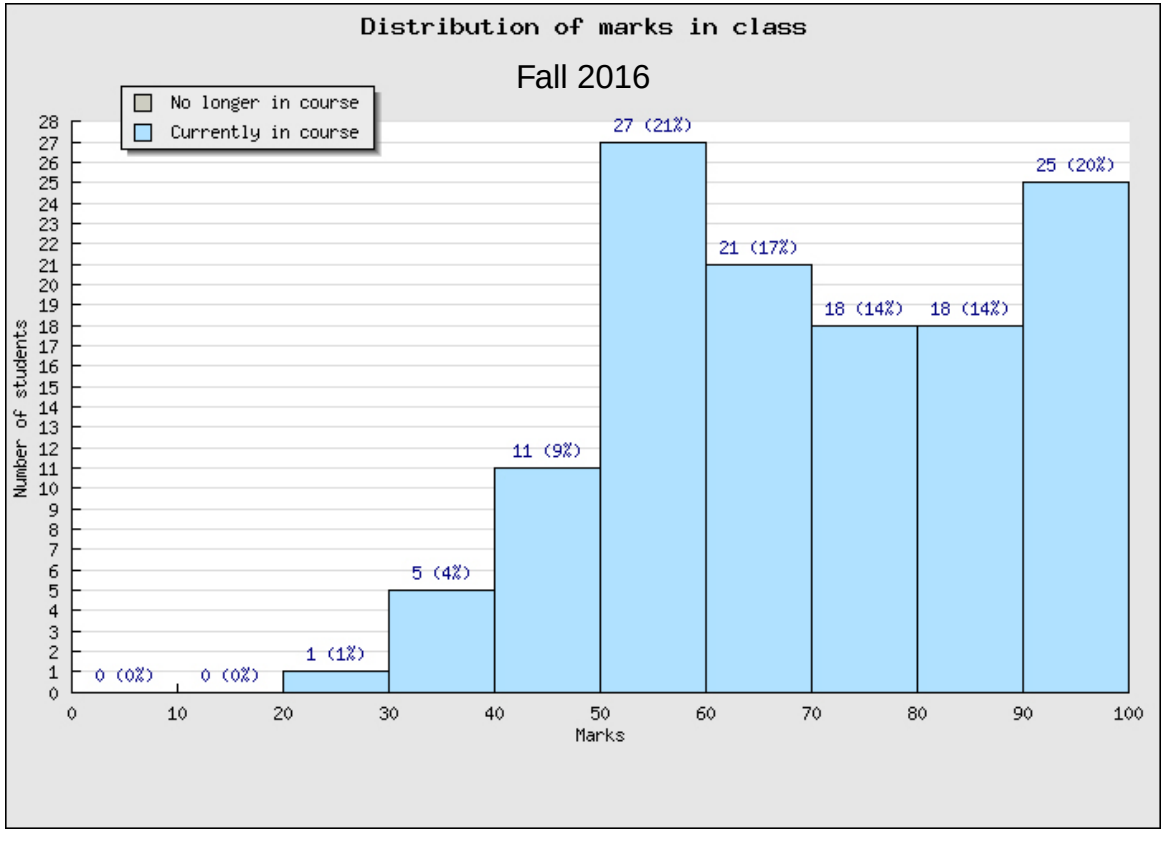

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  - sensible in an objective sense

Rational decision-maker

Model

A decision problem consists of

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A decision problem consists of

- a set A of possible *actions*

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Theory

Decision-maker chooses the member of A that is best according to her preferences

Many decision-makers: Strategic games

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any decision-making entity: individual human being, group of individuals, animal, . . .

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any set (numbers, lists of numbers, functions, . . .)

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action profile = list of actions, one for each player

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Preferences over action profiles $\Longrightarrow$ each player cares about actions taken by *other* players (as well as her own action)

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Strategic game: Example

- Players: two firms

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- (Other patterns are also possible)

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Working with preferences

Use numbers to represent them:

		Firm 2: *high*	Firm 2: *low*
Firm 1	*high*	2	0
	low	3	1

Firm 1's payoffs

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Working with preferences

Use numbers to represent them:

Firm 1's payoffs

Firm 1 \ Firm 2	*high*	*low*
high	2	0
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Firm 2's payoffs

Firm 1 \ Firm 2	*high*	*low*
high	2	3
low	0	1

Strategic game: Example

- Players: two firms
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Working with preferences

Combine tables:

		Firm 2: *high*	Firm 2: *low*
Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

Payoff of firm 1, payoff of firm 2

Strategic game: Example

		Firm 2	
		high	*low*
Firm 1	*high*	$2, 2$	$0, 3$
	low	$3, 0$	$1, 1$

Strategic game: Example

		Firm 2	
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Notes

- We could use other numbers to represent preferences (e.g. profits)

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- We could use other numbers to represent preferences (e.g. profits)
- For current purposes, only *order* matters
- What defines game are players' *preferences*, not specific payoff representations

Strategic game: Example

		Firm 2	
		high	*low*
Firm 1	*high*	$2, 2$	$0, 3$
	low	$3, 0$	$1, 1$

Same game:

		Firm 2	
		high	*low*
Firm 1	*high*	$100, 100$	$-5, 101$
	low	$101, -5$	$0, 0$

Strategic game: Example

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		high	*low*
Firm 1	*high*	$2, 2$	$0, 3$
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Same game:

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		high	*low*
Firm 1	*high*	$100, 100$	$-5, 101$
	low	$101, -5$	$0, 0$

		Firm 2	
		high	*low*
Firm 1	*high*	$0, 90$	$-100, 100$
	low	$100, -5$	$-1, 1$

Strategic game: Example

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Firm 1	*high*	$2, 2$	$0, 3$
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		Firm 2	
		high	*low*
Firm 1	*high*	$0, 90$	$-100, 100$
	low	$100, -5$	$-1, 1$

A game is determined by the players' *orderings* of the outcomes

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Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

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- Game is called

Strategic game: Example

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- Game is called *Prisoner's Dilemma*

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		high	*low*
Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

- Game is called *Prisoner's Dilemma*
- Structure of incentives in game is present in many situations

Strategic game: Example

		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
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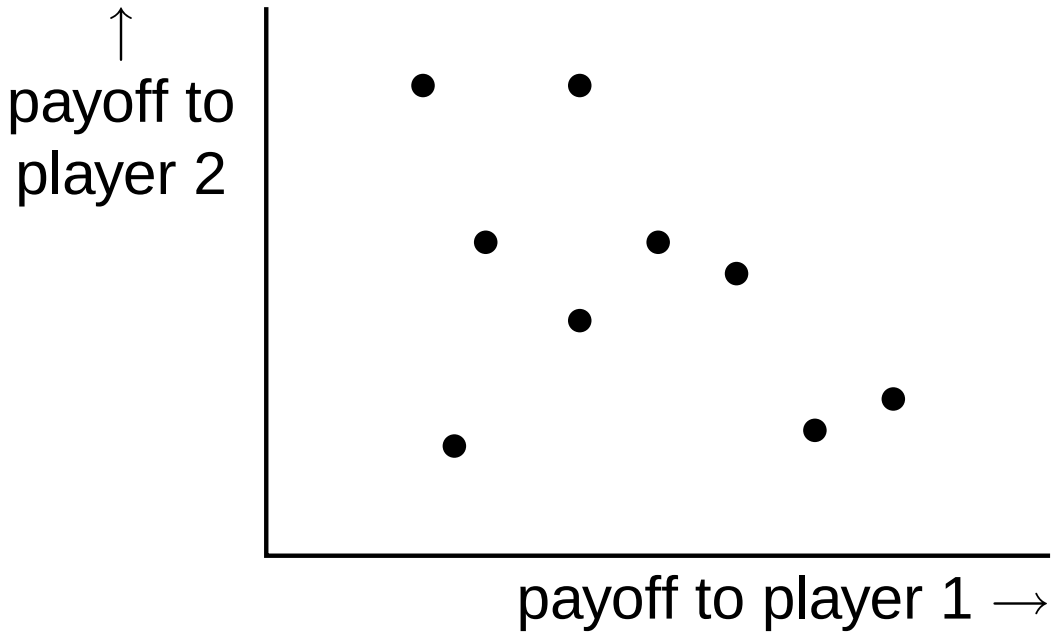
- Game is called *Prisoner's Dilemma*
- Structure of incentives in game is present in many situations
- Has been used to model a huge variety of situations in diverse fields

Digression: Pareto efficiency



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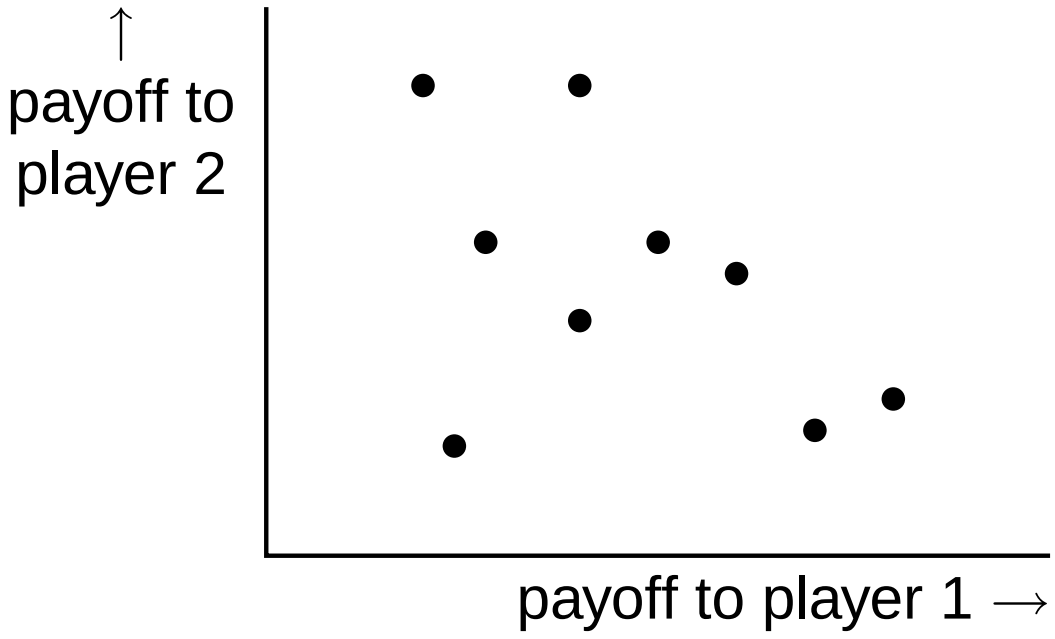
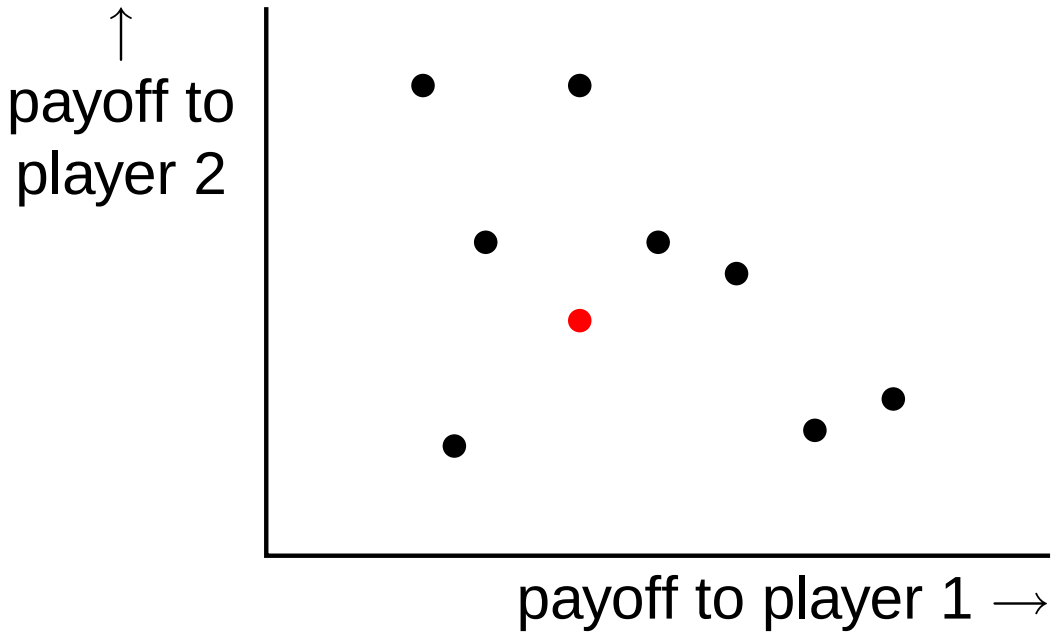


- Which outcomes are Pareto efficient?

Digression: Pareto efficiency



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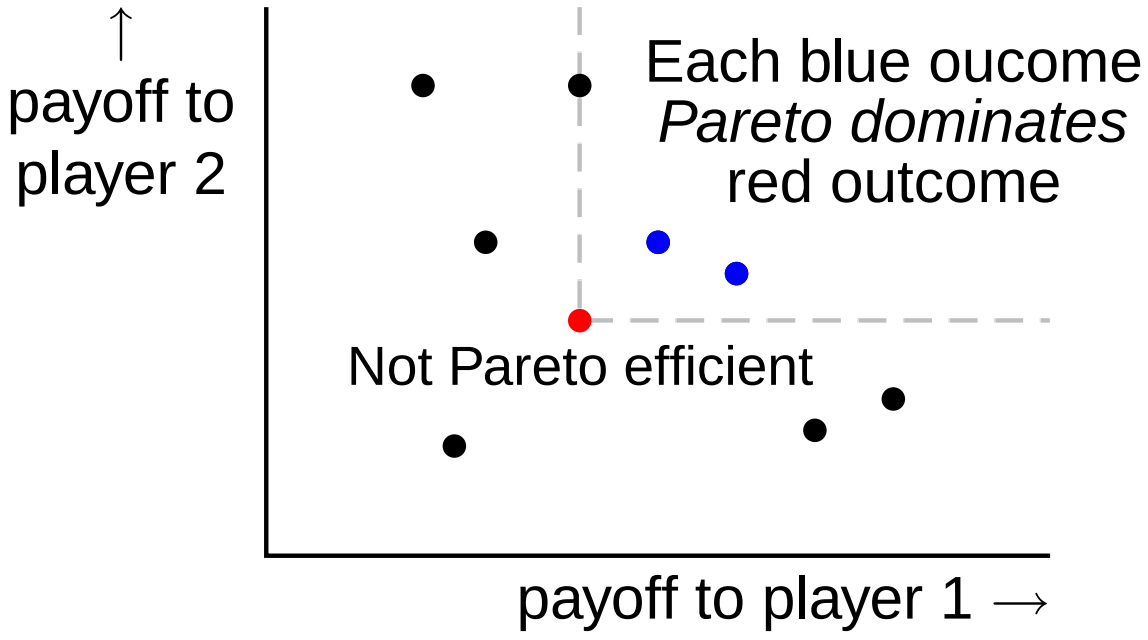

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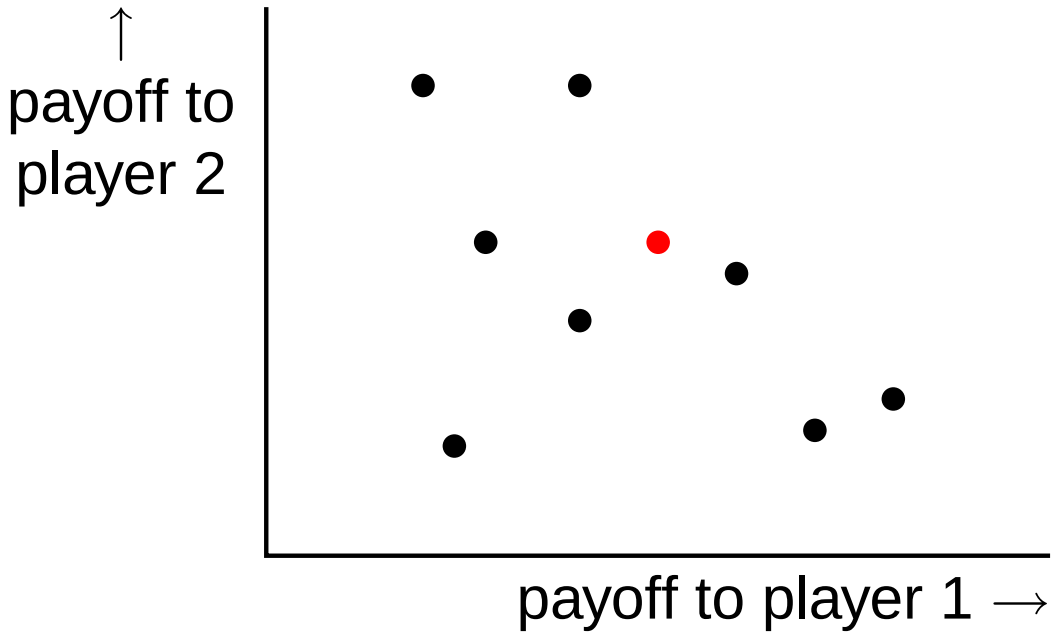

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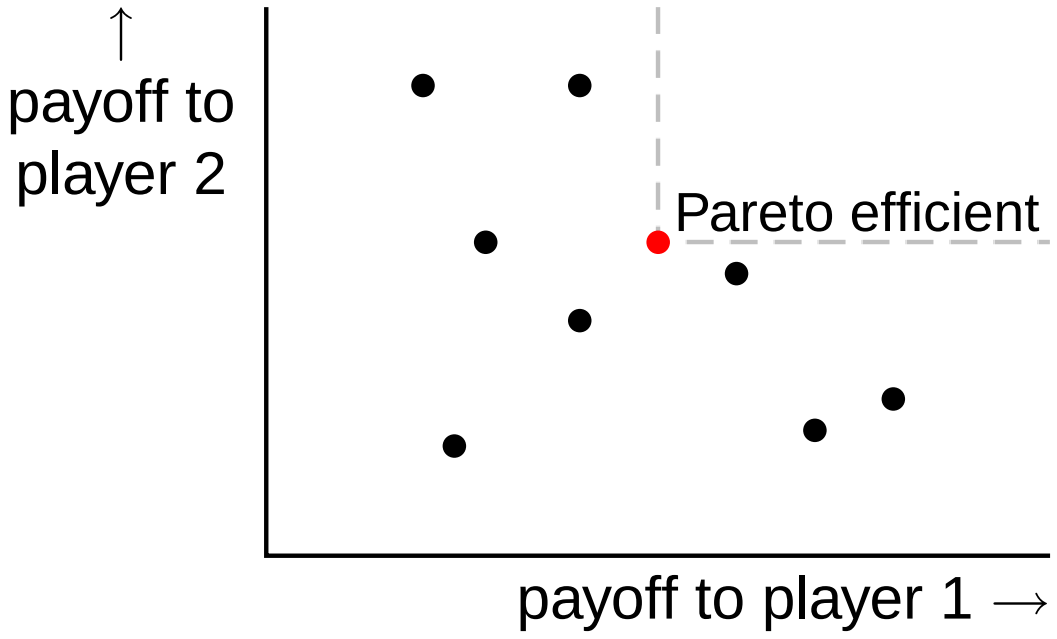

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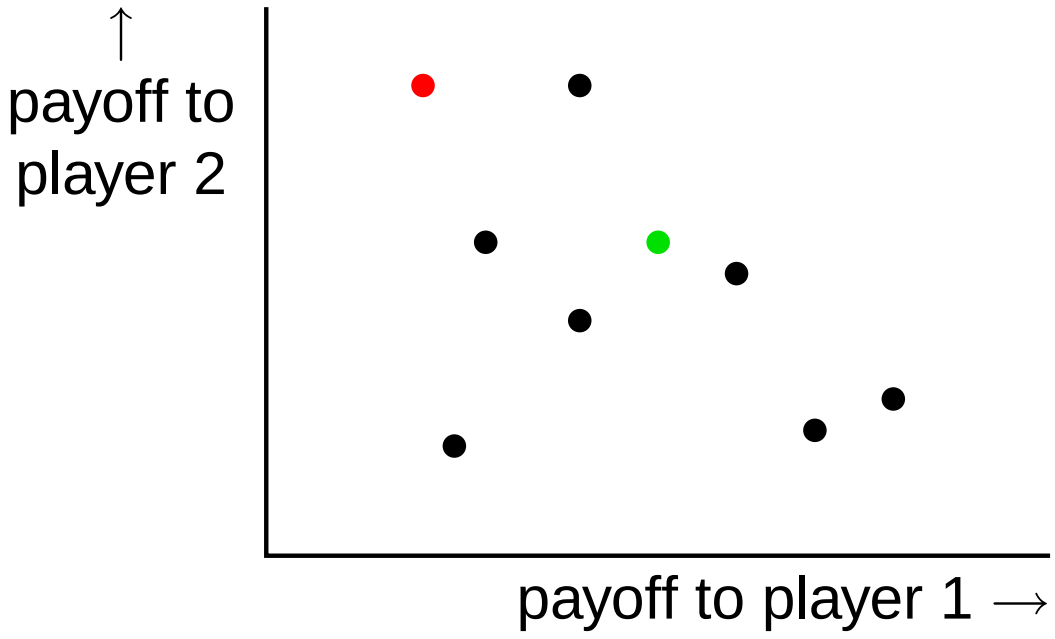

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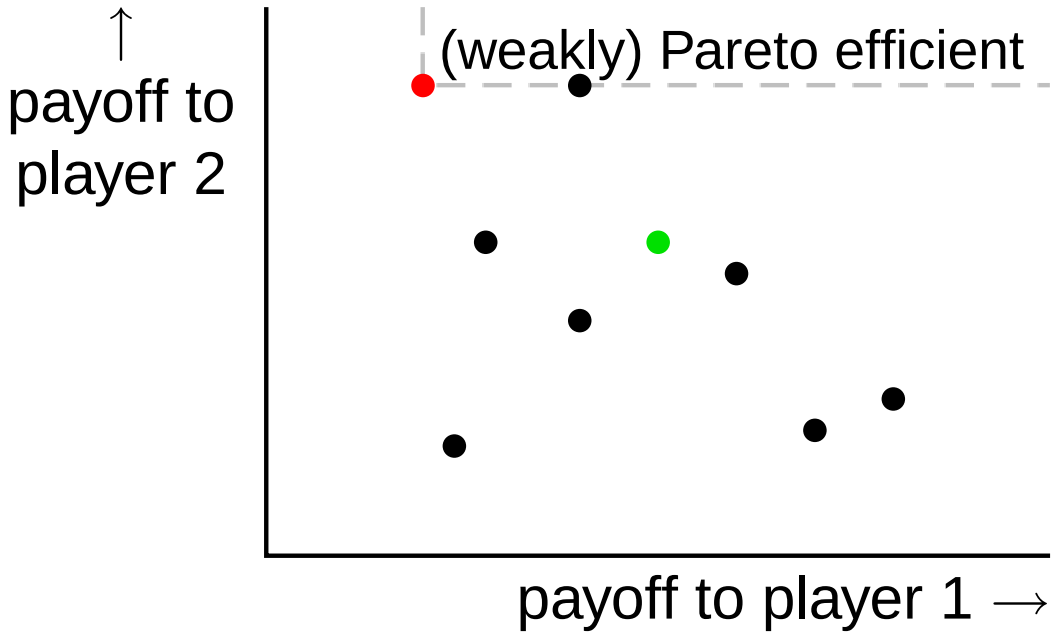

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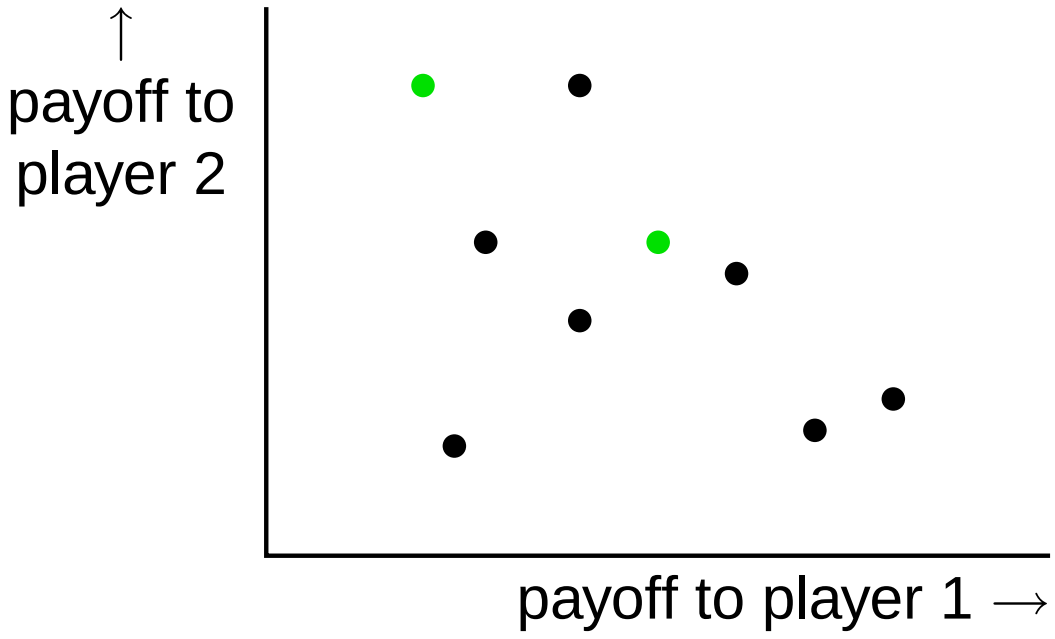

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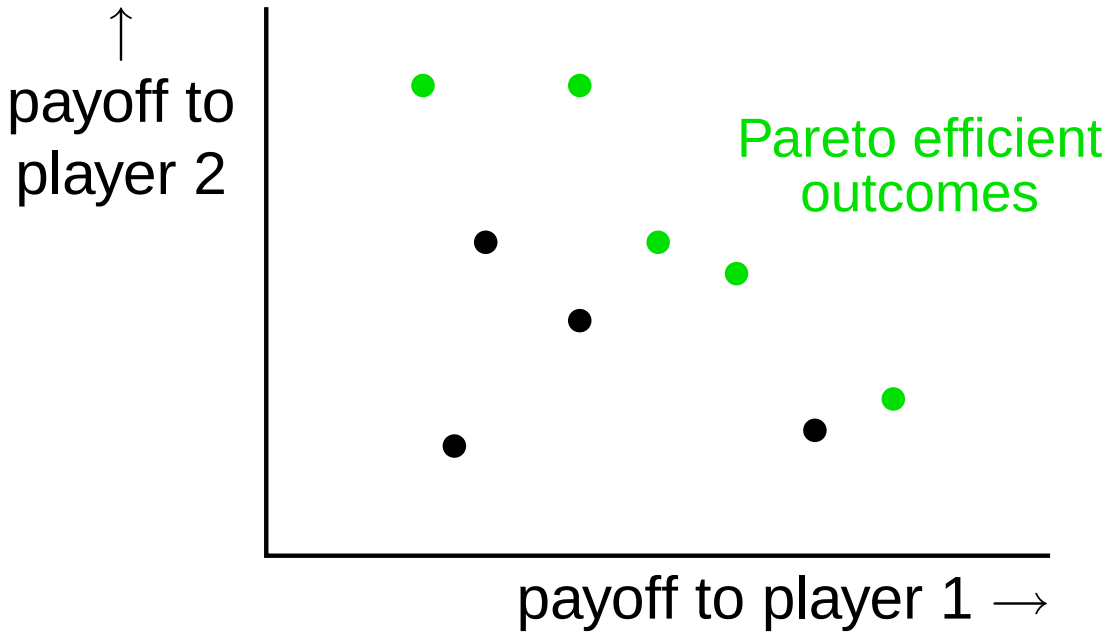

- Which outcomes are Pareto efficient?
- An outcome *a* is (weakly) *Pareto efficient* if there is no outcome that *every* player prefers to *a*

Strategic game: Prisoner's Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

- Which outcomes (action pairs) are Pareto efficient?

Strategic game: Prisoner's Dilemma

		Firm 2	
		high	*low*
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- Which outcomes (action pairs) are Pareto efficient?

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	low	3, 0	1, 1

- Which outcomes (action pairs) are Pareto efficient?
- $(high, high)$

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		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

- Which outcomes (action pairs) are Pareto efficient?
- $(high, high)$, $(high, low)$

Strategic game: Prisoner's Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

- Which outcomes (action pairs) are Pareto efficient?
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Strategic game: Prisoner's Dilemma

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Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

- Which outcomes (action pairs) are Pareto efficient?
- $(high, high)$, $(high, low)$, $(low, high)$

Strategic games: More examples

In other examples, the pattern of incentives is different

	X	Y
X	$2, 2$	$1, 1$
Y	$1, 1$	$0, 0$

Strategic games: More examples

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X	$2, 2$	$1, 1$
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X	$2, 2$	$0, 0$
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Coordination game

Strategic games: More examples

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Coordination game

	B	
B		

Bach

Strategic games: More examples

In other examples, the pattern of incentives is different

	X	Y
X	$2, 2$	$1, 1$
Y	$1, 1$	$0, 0$

	X	Y
X	$2, 2$	$0, 0$
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Coordination game

	B	
B		

Bach

Strategic games: More examples

In other examples, the pattern of incentives is different

	X	Y
X	$2, 2$	$1, 1$
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	X	Y
X	$2, 2$	$0, 0$
Y	$0, 0$	$1, 1$

Coordination game

	B	
B		

Bach

Strategic games: More examples

In other examples, the pattern of incentives is different

	X	*Y*
X	2, 2	1, 1
Y	1, 1	0, 0

	X	*Y*
X	2, 2	0, 0
Y	0, 0	1, 1

Coordination game

	B	*S*
B		
S		

Bach or Stravinsky?

Strategic games: More examples

In other examples, the pattern of incentives is different

	X	Y
X	$2, 2$	$1, 1$
Y	$1, 1$	$0, 0$

	X	Y
X	$2, 2$	$0, 0$
Y	$0, 0$	$1, 1$

Coordination game

	B	S
B	$2, 1$	
S		

Bach or Stravinsky?

Strategic games: More examples

In other examples, the pattern of incentives is different

	X	Y
X	$2, 2$	$1, 1$
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	X	Y
X	$2, 2$	$0, 0$
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Coordination game

	B	S
B	$2, 1$	
S		$1, 2$

Bach or Stravinsky?

Strategic games: More examples

In other examples, the pattern of incentives is different

	X	Y
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Coordination game

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Y	$0, 0$	$1, 1$

Coordination game

	B	S
B	$2, 1$	$0, 0$
S	$0, 0$	$1, 2$

Bach or Stravinsky?

	X	Y
X	$1, -1$	$-1, 1$
Y	$-1, 1$	$1, -1$

Matching pennies

A richer example

In examples so far, only two players, each with only two actions

A richer example

In examples so far, only two players, each with only two actions

Cournot's oligopoly game

A richer example

In examples so far, only two players, each with only two actions

Cournot's oligopoly game

- Players:

A richer example

In examples so far, only two players, each with only two actions

Cournot's oligopoly game

- Players: n firms

A richer example

In examples so far, only two players, each with only two actions

Cournot's oligopoly game

- Players: n firms
- For each firm
  - possible actions:

A richer example

In examples so far, only two players, each with only two actions

Cournot's oligopoly game

- Players: n firms
- For each firm
  - possible actions: outputs (nonnegative numbers)

A richer example

In examples so far, only two players, each with only two actions

Cournot's oligopoly game

- Players: n firms
- For each firm
  - possible actions: outputs (nonnegative numbers)
  - payoff:

A richer example

In examples so far, only two players, each with only two actions

Cournot's oligopoly game

- Players: n firms
- For each firm
  - possible actions: outputs (nonnegative numbers)
  - payoff: profit

A richer example

In examples so far, only two players, each with only two actions

Cournot's oligopoly game

- Players: n firms
- For each firm
  - possible actions: outputs (nonnegative numbers)
  - payoff: profit

Notes

- Many players, each with continuum of actions

A richer example

In examples so far, only two players, each with only two actions

Cournot's oligopoly game

- Players: *n* firms
- For each firm
  - possible actions: outputs (nonnegative numbers)
  - payoff: profit

Notes

- Many players, each with continuum of actions
- *Cannot* represent game in a table

Equilibrium outcomes

- We want to assume each player is rational

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- But each player doesn't know what others will do—so how to choose action?

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- Where do beliefs come from?

Equilibrium outcomes

- We want to assume each player is rational
- But each player doesn't know what others will do—so how to choose action?
- Form *beliefs* about others' actions
- Where do beliefs come from?
- Assume players have experience playing the game, or similar games—in fact, assume that their beliefs are *correct*

Equilibrium outcomes

- Each player's action is optimal given her beliefs

Equilibrium outcomes

- Each player's action is optimal given her beliefs
- Each player's belief is correct

Equilibrium outcomes

- Each player's action is optimal given her beliefs
- Each player's belief is correct

$\Rightarrow$ each player's action is optimal given other players' actions

Equilibrium outcomes

- Each player's action is optimal given her beliefs
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$\Rightarrow$ each player's action is optimal given other players' actions

Definition

A Nash equilibrium of a strategic game is an action profile

Equilibrium outcomes

- Each player's action is optimal given her beliefs
- Each player's belief is correct

$\Rightarrow$ each player's action is optimal given other players' actions

Definition

A Nash equilibrium of a strategic game is an action profile

list of actions, one for each player

Equilibrium outcomes

- Each player's action is optimal given her beliefs
- Each player's belief is correct

$\Rightarrow$ each player's action is optimal given other players' actions

Definition

A Nash equilibrium of a strategic game is an action profile with the property that every player's action is optimal

Equilibrium outcomes

- Each player's action is optimal given her beliefs
- Each player's belief is correct

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Definition

A Nash equilibrium of a strategic game is an action profile with the property that every player's action is optimal, given the other players' actions.

Equilibrium outcomes

- Each player's action is optimal given her beliefs
- Each player's belief is correct

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Definition

A Nash equilibrium of a strategic game is an action profile with the property that every player's action is optimal, given the other players' actions.

Equilibrium outcomes

- Each player's action is optimal given her beliefs
- Each player's belief is correct

$\Rightarrow$ each player's action is optimal given other players' actions

Definition

A Nash equilibrium of a strategic game is an action profile with the property that every player's action is optimal, given the other players' actions.

Equivalently: an action profile is a Nash equilibrium if no player can increase her payoff by changing her action, given the other players' actions

Prisoner's Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

Check each action pair in turn

Prisoner's Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	$2, 2$	$0, 3$
	low	$3, 0$	$1, 1$

Check each action pair in turn

- $(high, high)$:

Prisoner's Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	$2, 2$	$0, 3$
	low	$3, 0$	$1, 1$

Check each action pair in turn

- $(high, high)$: not a Nash equilibrium because firm 1 is better off deviating to *low* (and firm 2 is also better off deviating to *low*)

Prisoner's Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
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Check each action pair in turn

- (*high*, *high*): not a Nash equilibrium because firm 1 is better off deviating to *low* (and firm 2 is also better off deviating to *low*)
- (*high*, *low*):

Prisoner's Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
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Check each action pair in turn

- (*high*, *high*): not a Nash equilibrium because firm 1 is better off deviating to *low* (and firm 2 is also better off deviating to *low*)
- (*high*, *low*): not a Nash equilibrium because . . . .

Prisoner's Dilemma

		Firm 2	
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Check each action pair in turn

- (*high*, *high*): not a Nash equilibrium because firm 1 is better off deviating to *low* (and firm 2 is also better off deviating to *low*)
- (*high*, *low*): not a Nash equilibrium because . . . .
- (*low*, *high*):

Prisoner's Dilemma

		Firm 2	
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Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

Check each action pair in turn

- (*high*, *high*): not a Nash equilibrium because firm 1 is better off deviating to *low* (and firm 2 is also better off deviating to *low*)
- (*high*, *low*): not a Nash equilibrium because . . . .
- (*low*, *high*): not a Nash equilibrium because . . . .

Prisoner's Dilemma

		Firm 2	
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Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

Check each action pair in turn

- (*high*, *high*): not a Nash equilibrium because firm 1 is better off deviating to *low* (and firm 2 is also better off deviating to *low*)
- (*high*, *low*): not a Nash equilibrium because . . . .
- (*low*, *high*): not a Nash equilibrium because . . . .
- (*low*, *low*):

Prisoner's Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

Check each action pair in turn

- (*high*, *high*): not a Nash equilibrium because firm 1 is better off deviating to *low* (and firm 2 is also better off deviating to *low*)
- (*high*, *low*): not a Nash equilibrium because . . . .
- (*low*, *high*): not a Nash equilibrium because . . . .
- (*low*, *low*): Nash equilibrium because each player is worse off switching to *high* if other player's action is *low*.

Prisoner's Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

Check each action pair in turn

- (*high*, *high*): not a Nash equilibrium because firm 1 is better off deviating to *low* (and firm 2 is also better off deviating to *low*)
- (*high*, *low*): not a Nash equilibrium because . . . .
- (*low*, *high*): not a Nash equilibrium because . . . .
- (*low*, *low*): Nash equilibrium because each player is worse off switching to *high* if other player's action is *low*.

So: unique Nash equilibrium, (*low*, *low*).

Prisoner's Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

- Which outcomes (action pairs) are Pareto efficient?

Prisoner's Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

- Which outcomes (action pairs) are Pareto efficient?
  - $(low, high)$, $(high, high)$, $(high, low)$

Prisoner's Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

- Which outcomes (action pairs) are Pareto efficient?
  - $(low, high)$, $(high, high)$, $(high, low)$
- Note that the unique Nash equilibrium, (low, low), is *not* Pareto efficient

Prisoner's Dilemma

		Firm 2	
		high	*low*
Firm 1	*high*	2, 2	0, 3
	low	3, 0	1, 1

- Which outcomes (action pairs) are Pareto efficient?
  - $(low, high)$, $(high, high)$, $(high, low)$
- Note that the unique Nash equilibrium, (low, low), is *not* Pareto efficient

Split or steal?

Split or steal?

	Split	*Steal*
Split	50,075, 50,075	0, 100,150
Steal	100,150, 0	0, 0

Split or steal?

	Split	*Steal*
Split	50,075, 50,075	0, 100,150
Steal	100,150, 0	0, 0

Nash equilibria?

Split or steal?

	Split	*Steal*
Split	50,075, 50,075	0, 100,150
Steal	100,150, 0	0, 0

Nash equilibria? $(Split, Steal)$

Split or steal?

	Split	*Steal*
Split	50,075, 50,075	0, 100,150
Steal	100,150, 0	0, 0

Nash equilibria? $(Split, Steal)$, $(Steal, Split)$

Split or steal?

	Split	*Steal*
Split	50,075, 50,075	0, 100,150
Steal	100,150, 0	0, 0

Nash equilibria? $(Split, Steal)$, $(Steal, Split)$, and $(Steal, Steal)$

Split or steal?

	Split	*Steal*
Split	50,075, 50,075	0, 100,150
Steal	100,150, 0	0, 0

Nash equilibria? $(Split, Steal)$, $(Steal, Split)$, and $(Steal, Steal)$

Alternative representation:

	Split	*Steal*
Split	1, 1	0, 2
Steal	2, 0	0, 0

Split or steal?

	Split	*Steal*
Split	50,075, 50,075	0, 100,150
Steal	100,150, 0	0, 0

Nash equilibria? $(Split, Steal)$, $(Steal, Split)$, and $(Steal, Steal)$

Alternative representation:

	Split	*Steal*
Split	1, 1	0, 2
Steal	2, 0	0, 0

Compare with *Prisoner's Dilemma*:

	X	*Y*
X	2, 2	0, 3
Y	3, 0	1, 1

Only difference between games:

Split or steal?

	Split	*Steal*
Split	50,075, 50,075	0, 100,150
Steal	100,150, 0	0, 0

Nash equilibria? $(Split, Steal)$, $(Steal, Split)$, and $(Steal, Steal)$

Alternative representation:

	Split	*Steal*
Split	1, 1	0, 2
Steal	2, 0	0, 0

Compare with *Prisoner's Dilemma*:

	X	*Y*
X	2, 2	0, 3
Y	3, 0	1, 1

Only difference between games: indicated preferences

Split or steal?

	Split	*Steal*
Split	50,075, 50,075	0, 100,150
Steal	100,150, 0	0, 0

Nash equilibria? (*Split*, *Steal*), (*Steal*, *Split*), and (*Steal*, *Steal*)

Alternative representation:

	Split	*Steal*
Split	1, 1	0, 2
Steal	2, 0	0, 0

Compare with *Prisoner's Dilemma*:

	X	*Y*
X	2, 2	0, 3
Y	3, 0	1, 1

Only difference between games: indicated preferences

Example of Nash equilibrium: Coordination game

	X	Y
X	$2, 2$	$0, 0$
Y	$0, 0$	$1, 1$

Example of Nash equilibrium: Coordination game

	X	Y
X	$2, 2$	$0, 0$
Y	$0, 0$	$1, 1$

Two Nash equilibria, (X, X) and (Y, Y)

Example of Nash equilibrium: *Bach or Stravinsky?*

	Bach	*Stravinsky*
Bach	$2, 1$	$0, 0$
Stravinsky	$0, 0$	$1, 2$

Example of Nash equilibrium: *Bach or Stravinsky?*

	Bach	*Stravinsky*
Bach	$2, 1$	$0, 0$
Stravinsky	$0, 0$	$1, 2$

Two Nash equilibria, (B, B) and (S, S)

Example of Nash equilibrium: *Matching Pennies*

	H	T
H	$1, -1$	$-1, 1$
T	$-1, 1$	$1, -1$

Example of Nash equilibrium: *Matching Pennies*

	H	T
H	$1, -1$	$-1, 1$
T	$-1, 1$	$1, -1$

No Nash equilibrium!

Example of Nash equilibrium

	L	R
T	$1, 1$	$2, 1$
B	$0, 0$	$2, 4$

Example of Nash equilibrium

	L	R
T	$1, 1$	$2, 1$
B	$0, 0$	$2, 4$

Nash equilibria: (T, L), (T, R), and (B, R)

Example: Investing in a joint project

- n people

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- So for every person,

  invest & project succeeds $\succ$ don't invest $\succ$ invest & project fails

Investing in a joint project

Strategic game

- Players:

Investing in a joint project

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If player chooses *Don't invest*, 0 regardless of others' actions

Investing in a joint project

Nash equilibrium

- k people invest?

Investing in a joint project

Nash equilibrium

- k people invest?
- n people invest?

Investing in a joint project

Nash equilibrium

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Investing in a joint project

Nash equilibrium

- k people invest?
- n people invest?
- no one invests?
- some other number of people invest?

Investing in a joint project

Reminder of payoffs:

- *Invest* $\Rightarrow$ 100 if $\geq k$ investors, -10 if $< k$ investors
- *Don't invest* $\Rightarrow$ 0

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- between 1 and $k-1$ people invest: not Nash equilibrium because investor deviates $\Rightarrow$ gets 0 rather than -10

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Summary

Exactly two Nash equilibria:

- everyone invests
- no one invests

Traveler's Dilemma

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$$\begin{cases} \\ \\ \\ \end{cases}$$

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$$\begin{cases} \\ \\ \end{cases} \quad \text{if } a_i = a_j$$

where a_i is i's action and a_j is the other player's action

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$$\begin{cases} & \text{if } a_i < a_j \\ a_i & \text{if } a_i = a_j \\ & \end{cases}$$

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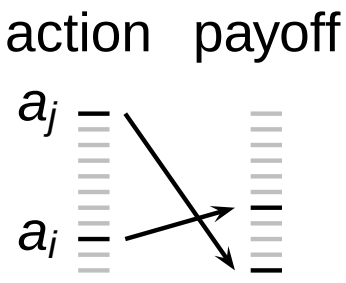
- $a_i < a_j$?



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- $a_i < a_j$?
  - Not NE: j lowers a_j to a_i
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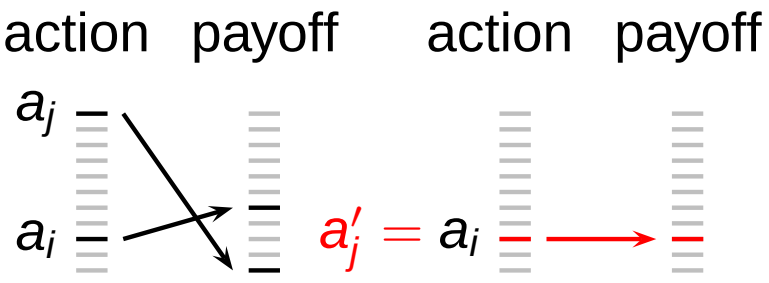

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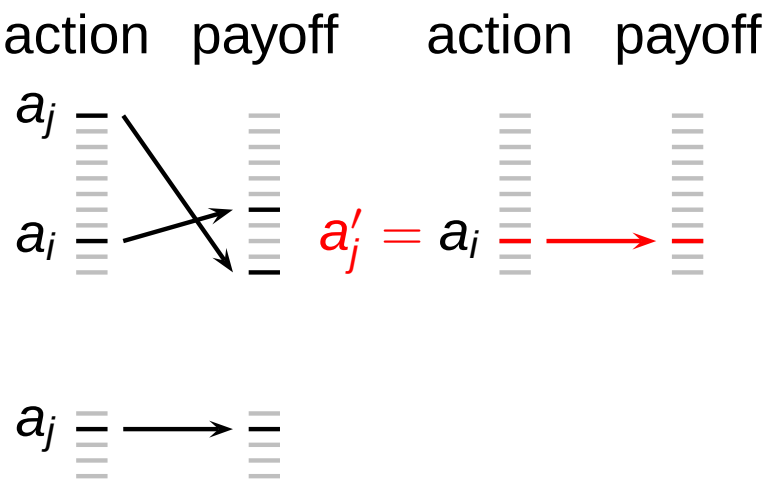

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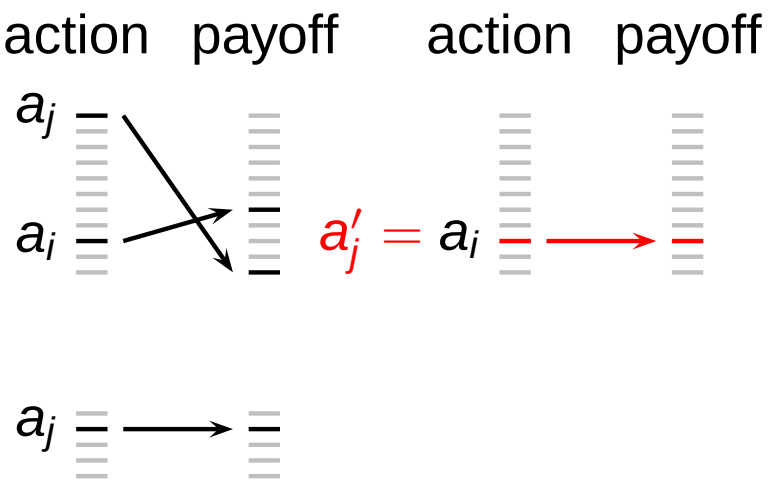
  - If $a_i \geq 3$



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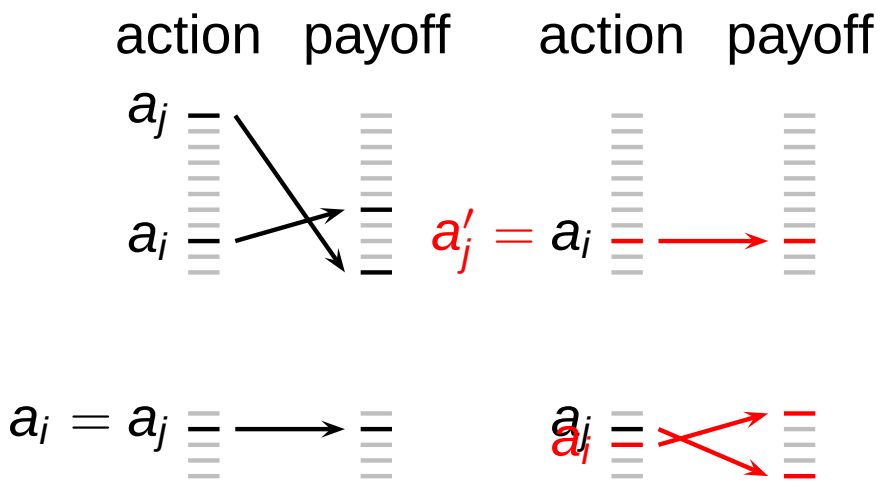

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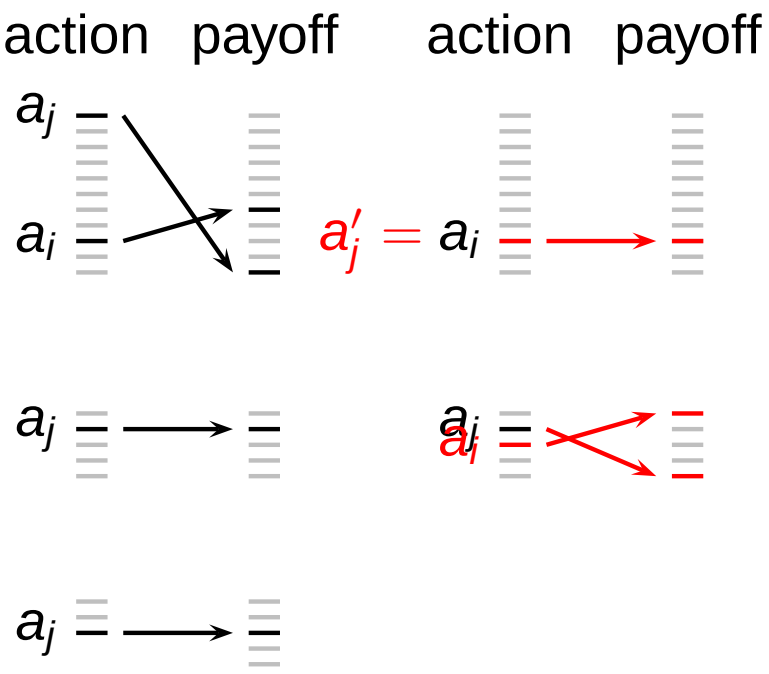
  - If $a_i = a_j = 2$



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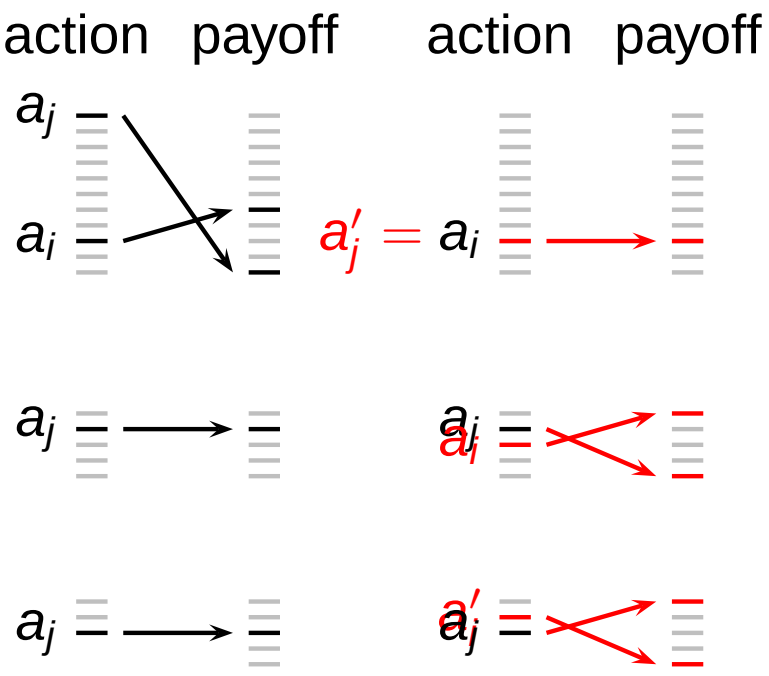
  - If $a_i \geq 3$, not NE: i lowers a_i to $a_i - 1 \Rightarrow$ increases i's payoff
  - If $a_i = a_j = 2$, NE! If either player increases amount, payoff $= 0$



Traveler's Dilemma

Summary

Unique Nash equilibrium: both travelers name the lowest possible valuation, $2